Probabilistic Assessment of the Mechanical Properties of a 3D-Braided Pultruded Fiber-Reinforced Polymer

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Abstract
This work describes research aimed to probabilistically assess the mechanical properties of a pultruded fiber-resin reinforced polymer (FRP) reinforced with a 3D braid and roving. In addition, reliability-based reduction factors for the design of such materials are developed and proposed. Moduli are investigated because the design of most civil engineering applications is dominated by serviceability and buckling limit states rather than the strength. To possess a certain level of confidence in these value estimation, it is necessary to fully explore the material applications. This methodology combines simplified classical lamination theory in one case and a 3D model (Fiber Inclination Model) for braid performance in another case with experimental values in a probabilistic model to simulate the randomness of the system by the Monte Carlo technique. The accuracy of the theoretical model is investigate, evaluating the coupling effect caused by the variations in constituents and fiber orientation. Randomness is considered beginning at the micromechanical level (fiber/matrix) up to the macromechanical level (ply mechanics). All fibers were E-glass embedded in vinyl ester matrix. Reduction factors for the material are suggested providing at least a 95% confidence level.

Keywords
Fiber-Reinforced Polymers, Probabilistic Analysis, Design Constants, 3D Braided Preform

1. Introduction
For the past 30 years, fiber-reinforced polymers (FRP) has gained popularity in many civil engineering applications. Their applications include structural shapes for buildings, bridges, reinforcement for concrete elements, rehabilitation and repair of structures, etc. (Barbero et al.,1991a, and Barbero et al.,1991b). These materials have superior characteristics compared to traditional construction materials. They can be specifically tailored to satisfy the stress condition of a particular application or can be generally tailored yet performing extraordinarily. In addition, they possess high strength to weight ratios, and they have high resistance to environmental degradation. Regardless of all the outstanding properties of these materials, their use is limited because of the lack of formal design codes. The manufacturing flexibility and the wide variety of manufacturers and manufacturing techniques make difficult to develop unified standards. Also, their design needs special attention to account for their low modulus and
ductility compared with traditional materials such as steel. This issue was raised at the workshop *Repairing and Rehabilitating the Buildings and Bridges of the Americas, Hemispheric Workshop on Future Directions* sponsored by the National Science Foundation and the University of Puerto Rico at Mayagüez, April 2001 (Wendishansky and Pumarada-O’Neill, 2002).

Several organizations and individual researchers have independently developed guidelines for structural design of FRP members and concrete reinforcement. Effort made in Japan, Canada, United States, and Europe to develop codes and standards for FRP materials in civil structures are summarized in Bakis, et al. (2002). Individual researchers reported simplified design approaches for FRP structural shapes. Two examples are Zureick and Steffen (2000) for angle sections, and Shekar et al. (2000) for FRP bridge deck.

The published properties of FRP are very scarce due to variations in properties of the fiber/matrix, amount of matrix voids, and fabrication parameters such as geometry and fiber orientation. At the structural level, the calculation of the structural response is affected by the scarcity at the material level combined with uncertainties of the loading and environmental conditions. Traditional analytical methods are deterministic, ignoring uncertainties inherent in the material behavior. Because the majority of the variations at the different levels are unknown, the traditional approaches rely heavily on safety factors to account for the scatter in the response. It is necessary to quantify the uncertainties at the different levels and compare it with real material and structural responses to develop design provisions based on fundamentals of probability and reliability.

Acosta (2002) proposed a series of reliability-based reduction factors for two types of FRP materials to be used in the design of FRP decks. These factors were developed from experimental data of coupons from groups of 5 to 10 specimen. To obtain valid statistical data, this study expands the work by Acosta (2002) including a total of 30 tensile coupons per material direction adding an alternate analytical model to redefine the design factors proposed by Acosta (2002). This paper presents this effort with a description of the material architecture, the methodology used, and the preliminary results from the analyses.

2. Material Description

The material used for this study was excise from a pultruded section of a vynilester resin reinforced with different configurations of E-glass fibers. Figure 1 shows a cross-section of the FRP perpendicular to the principal direction. The material is composed of two outer layers of CSM, followed by roving with a 3D braided preform at the middle for principal reinforcement. The 3D braided perform has straight strands running in the 0° direction and strands oriented in the +/- 70° and through the thickness. Tensile testing were conducted on coupon size samples from the material according to the ASTM standards and following recommendations by Wang and Zureick (1994), and by Masters and Portanove (1996). Coupons were 50 mm (2 in) wide based on recommendations from Acosta (2002) to reduce the variability induced by the uneven distribution of the roving as it can be seen in Figure 1. The material was tested along the x-direction, which is parallel to the roving. The y-direction is perpendicular to the x-direction.

![Figure 1: Representation of the Reinforcement 3-D Braid Scheme.](image-url)
3. Reliability-Based Model

The reliability model used to characterize the FRP is presented in Acosta (2002). It follows methodologies used by many authors including Mase, et al. (1991), Shiao, et al. (1993), Murthy, et al. (1997), and Chamis, et al. (1999). The model determine $\phi$ factors for the elastic moduli of the FRP considering variabilities of the material at different material and structural levels. The process goes from the material fundamental variables (i.e. fiber and matrix properties), and manufacturing variables (i.e. void content and fiber orientation). The moduli is expressed in the form of $M_{\text{pred}} = \phi M_{\text{th}}$, where $M_{\text{th}}$ is the modulus computed using the well known classical lamination theory and the fiber inclination model (Yang, et al, 1986). It is known that experimental results of modulus, $M_{\text{exp}}$, are random in nature and exhibit variabilities. If the randomness of the theoretical values is established, then the problem can be formulated in terms of a performance function, $g$, expressed as:

$$g = M_{\text{exp}} - M_{\text{pred}}$$

where $M_{\text{exp}}$ is taken as the descriptive statistic of experimental values of the modulus of interest. If $M_{\text{exp}}$ and $M_{\text{th}}$ are random, then $g$ is also random, as shown schematically in Figure 2. It is proposed that the value of $\phi$ be determined such that $g > 0$ with a probability of success $p_s \geq 95\%$. The shaded area in Figure 2 represents the probability of not meeting the requirement of $g > 0$.

![Figure 2: Probability Distribution of the Performance Function $g$.](image)

The theoretical engineering moduli were predicted by the macromechanical model for ply mechanics assuming no coupling between in-plane axial-shear deformations. The expressions are:

$$M_{\text{th}}^x = E_{\text{th}}^x = \frac{A_{11}A_{22} - A_{12}^2}{t_f A_{22}}$$

$$M_{\text{th}}^y = E_{\text{th}}^y = \frac{A_{11}A_{22} - A_{12}^2}{t_f A_{11}}$$

where the $A$’s are elements of the well known extensional stiffness matrix (Barbero, 1998). The basic lamina properties were in one case expressed by the classical lamination formulation and by the modified classical lamination known as the fiber inclination model (FIM) (Yang, 1986) in other case. FIM analyzes the unit cell structure of the 3D reinforcement as show in Figure 3. The compliance matrix $[Q]$ for the unit cell is computed and later incorporated with the other layer considered 2D to assemble the lamite to reach the expressions in Equations 2 and 3. The material properties used in this model are the
same used in the CLT. Coupling does not exist for balanced and symmetric laminates. However, due to
the probabilistic nature of the analysis, the condition changes inducing the coupling behavior. Equations
2 and 3 are functions of the fundamental material properties $E_f$, $E_m$ (fiber and matrix modulus), $V_f$, $V_m$
(fiber and matrix Poisson’s ratios), and $\rho_f$ (density of fibers), and the manufacturing variables $v_v$, $v_f$
( voids and fiber fractions), $\Theta$ (fiber orientation), $w_i$ (weight fractions), $w_{af}$ (weight per unit area of fabric), and $t_F$
(layer thickness). These variables were treated as random and were assumed to have normal distribution
although FRP most of the time follow Weibull distributions. Values for all the variables were either
taken from the literature or from the manufacturers and their variation is based on a coefficient of
variation of 5% (Shiao et al., 1993, Liaw et al., 1993).

The values of $\beta$ were computed using the Monte Carlo simulation. A comparison of the distribution
functions of the experimental values and the prediction from both models is shown in Figure 4. The great
variability of the experimental values is observed. There is less randomness in the FIM than in the CLT
for both cases. Higher randomness is observed in the modulus along the y-direction. A range of $p_s$ was
evaluated for $\phi$ factors ranging from 1.0 to 0.5 considering both models. Results from the analysis are
shown in Figure 5. The heavy horizontal line in the figure represents the level at which $p_s[i > 0] = 95%$.
From the graph in Figure 5, it is observed that the optimal $\phi$ factor for the modulus in the x-direction is
0.75 while for the y-direction is 0.5. In Acosta (2002), the suggested $\phi$ factor for the material was 0.55,
which was the lowest value from coupons tested in tension and compression varying the coupon widths
(Acosta-Costa, 1999).

![Figure 3: Unit Cell Structure Scheme and 3-D Braid Configuration](image)

![Figure 4: Comparison of the Statistical Distribution of the Theoretical Modulus and the Experimental Values.](image)
4. Conclusions

This paper presents an expansion of a work aimed to develop reliability-based reduction factors for the design of fiber-reinforced polymeric composites for civil engineering applications. The material used was a pultruded FRP reinforced with a 3D braided preform. A total of 30 coupons were tested in tension along the two principal material directions. This study considered material variabilities at both the fundamental material properties (fiber/matrix) and the manufacturing variables (i.e. fiber content, fiber orientation angle, and layer thicknesses). From the work it is concluded that:

- The new experimental data and models found $\phi$ factor of 0.75 for x-direction and 0.5 for y-direction. To be conservative, a value of 0.5 should be considered. This value is lower than the factors proposed by Acosta (2002).
- The variability used for the prediction was arbitrarily selected appearing to be smaller than the values obtained experimentally. Other manufacturing variables such as fiber breakage, voids, and dry fibers were not considered in the model and they will greatly increase the randomness of the FRP.
- The use of classical lamination theory and the rule of mixture as basis for the reliability model can represent the response of the plates and sections reinforced with the stitched fabrics. However, it is a reasonable approximation for the sections reinforced with the 3-D performs because it does not account for the sophisticated structure that it possesses.
- This work will be expanded for a wider range of material configurations, varying the fiber architecture, manufacturing technique, and structural shape.
- Material degradation models should be included in the study to determine the long-term effect. This is one of many issues that need to be resolved in the FRP industry.

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References


