

# **Chaos and Synchronization Through the Study of Two Coupled Chua's Circuits**

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## **ABSTRACT**

It is used the Chua circuit to demonstrate the possibility of obtaining synchronization of chaos with numerical simulations of a coupled nonlinear system consisting of two coupled non-identical circuits. Synchronization is demonstrated through a correlation diagram while the dynamic behavior is explained with a bifurcation diagram. One of the circuit has an additional noise source, while to the other has a periodic voltage source.

**Keywords:** Dynamical Systems, Bifurcation Theory, Chua's Circuit, synchronization

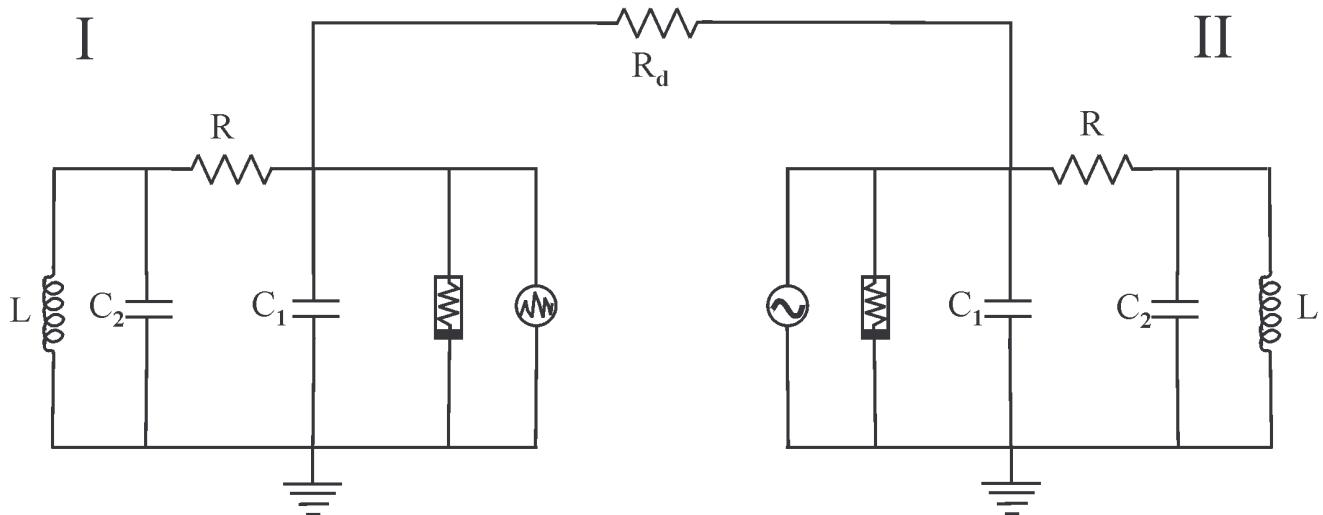
## **1. INTRODUCTION**

Chua's circuit is one of the most popular nonlinear dynamical system used to investigate the dynamic behavior in all possible states: Periodic, quasi-periodic and chaotic. It is very simple electronic circuit capable to exhibit a rich dynamical behavior. It was introduced in 1987 by Leon O. Chua (Chua and Desoer, 1987, Santoboni, 1999), since then, the ease of construction of the circuit has made it a natural real-world example of a chaotic system. It was the first physical implementation of chaos which has been rigorously proved (Kapitaniak, 2000). Any autonomous circuit made from standard components (resistors, capacitors, inductors) must satisfy three criteria in order to show chaotic behavior, i) At least one non-linear element, ii) one or more locally active resistors, iii) three or more energy-storage elements. Chua's circuit is the simplest electronic circuit meeting these criteria (Itoh and Hayashi, 1998). This circuit has been studied extensively since its initial proposal, and serves as a test platform for many research areas that involve bifurcation processes including chaos. (Calvo et al., 2001). The robustness, ease of implementation, and its nonlinear behavior contribute to the success between theoretical and experimental researchers of chaos theory. Chua's circuit can be implemented using a linear network connected to a nonlinear device such as a Chua's diode (Itoh, 2000).

This paper studies a system that uses a circuit that has been the primary reference for the study of chaos in electronic systems over the past ten years.

## **2. BASICS CONCEPTS**

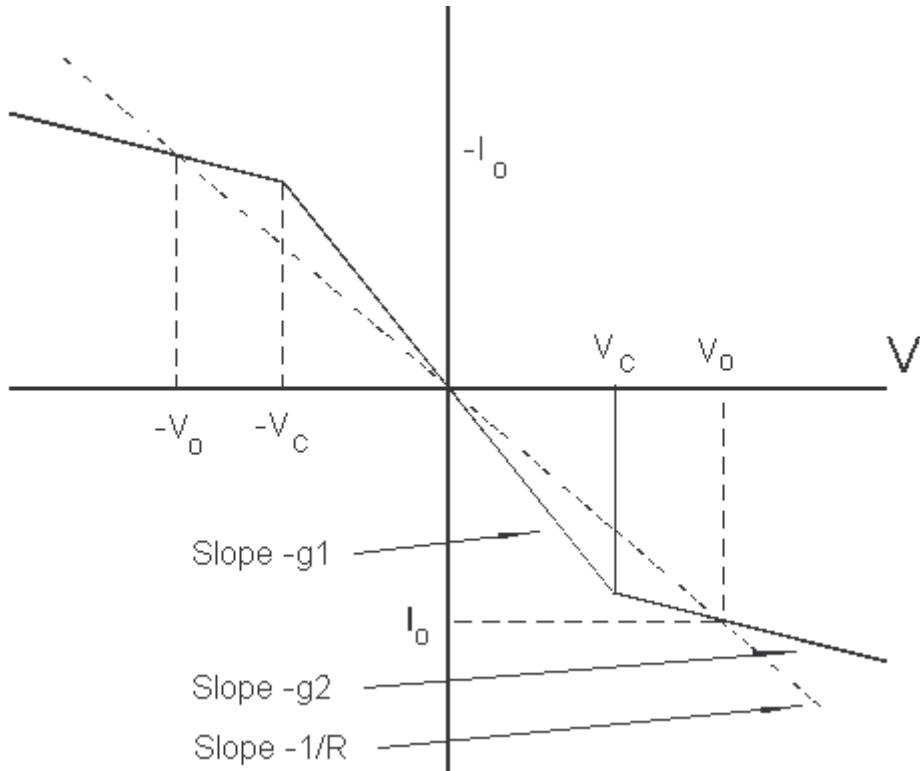
Word chaos normally means that a predictable dynamic system can give unpredictable results. The easiest way to observe chaos is in electronic circuits. This is because of its simplicity, inexpensive and because electronic devices are well understood, so Chua's circuit is an example of a chaotic circuit (Madam, 1993). But because of its simplicity and universality, this circuit is bit more special. A lot of questions can be asked about this system. In this work an answer will be given to the question whether it is possible to synchronize two Chua's circuits. The two Chua's circuits will have different starting values and/or different values for the components. First there will be looked at the onset of chaos. When it is determined how the circuit works, there will be explained to what extend the circuit is controlled and how it can be synchronized. The resistive coupling of two Chua's circuits may be done as in figure 1 (Avendaño, 1995).



**Figure 1:** System consisting of two Chua's circuits coupled together. Each circuit has an additional perturbation source, the on left-hand side, the circuit-I has an extra source noise, whereas on the other side the circuit-II has an extra sinusoidal source.

How the circuit works. The circuit can be broken down into three parts: The inductor  $L$  (together with its associated resistance  $r$ ), the resistor  $R_d$ , and the capacitors  $C_1$  and  $C_2$  make up a conventional linear oscillation circuit. With the rest of the circuit shorted out, this part would give damped oscillations. The operational amplifier associated to resistors have the effect of a negative resistance of size  $-R_1$ . This is still a linear circuit element - it does not by itself give chaos, although it acts as the source of energy for the dynamics. The diode pair gives the nonlinearity in the circuit. The diodes simply serve to switch in the resistance  $R_2$  in parallel with the resistance  $-R_1$  when the voltage gets larger than the switch on voltage. Figure 2 shows the behavior of different slopes associated with nonlinear resistors in the system. The nonlinear characteristic curve is segmented into three linear parts with negative slopes.

Transfer characteristic of Chua's diode. When a two-terminal resistor is characterized by a curve other than a straight line through the origin, it is called a nonlinear resistor or diode Chua (Kapitaniak et al., 1994, Palenzuela et al., 2001). There are two types of diodes Chua: a) A voltage controlled Chua diode, and b) A current controlled Chua diode. So, Chua diodes can be considered as current sources controlled by voltage type (a) or as sources of current-controlled for the type (b).



**Figure 2: Transfer function of the nonlinear impedance for the Chua's Circuit.**

### 3. THE MODEL

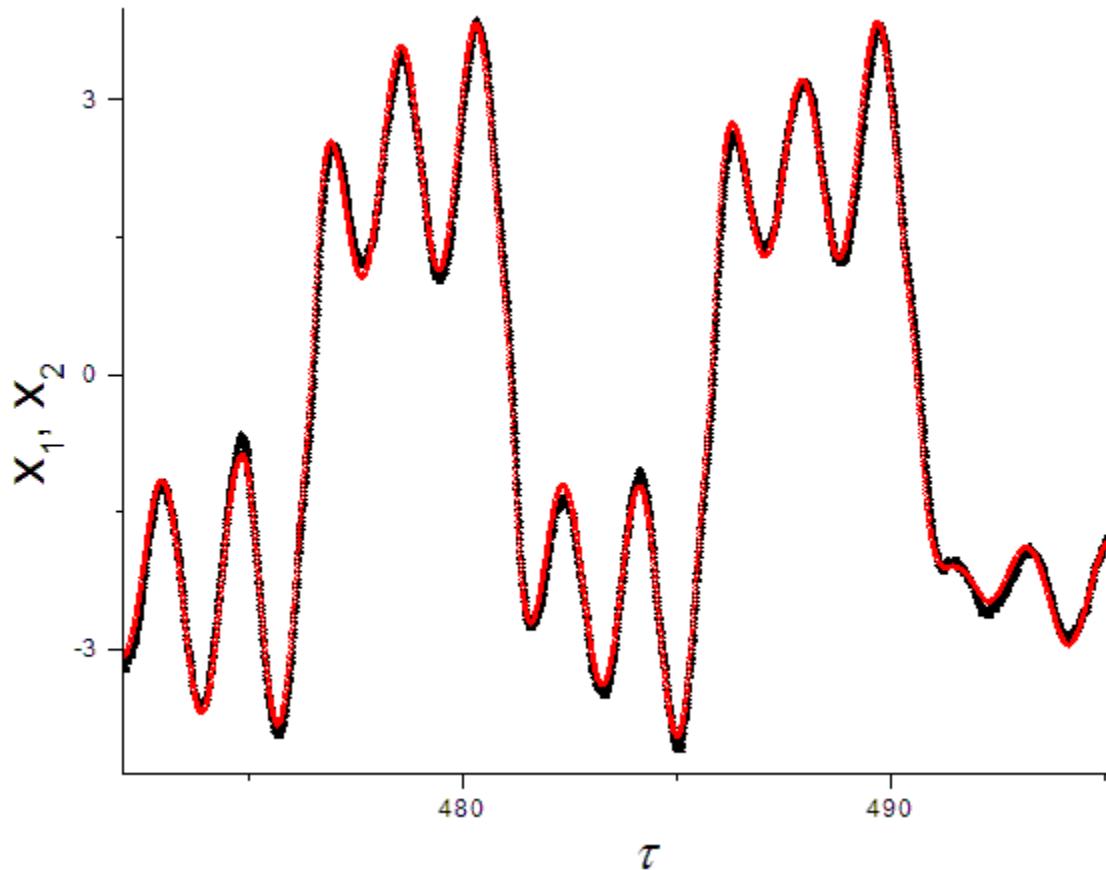
The model of two identical Chua's Circuits coupled with a resistive element connected to both positive point of the closest capacitor of the negative resistance element, with bidirectional communication in normalized units is (Itoh, 1997):

$$\begin{aligned}
 \frac{dx_1}{d\tau} &= \alpha[y_1 - x_1 - f(x_1) - \eta N(0,1)] + \delta(x_2 - x_1) \\
 \frac{dy_1}{d\tau} &= x_1 - y_1 + z_1 \\
 \frac{dz_1}{d\tau} &= -\beta y_1
 \end{aligned} \tag{1}$$

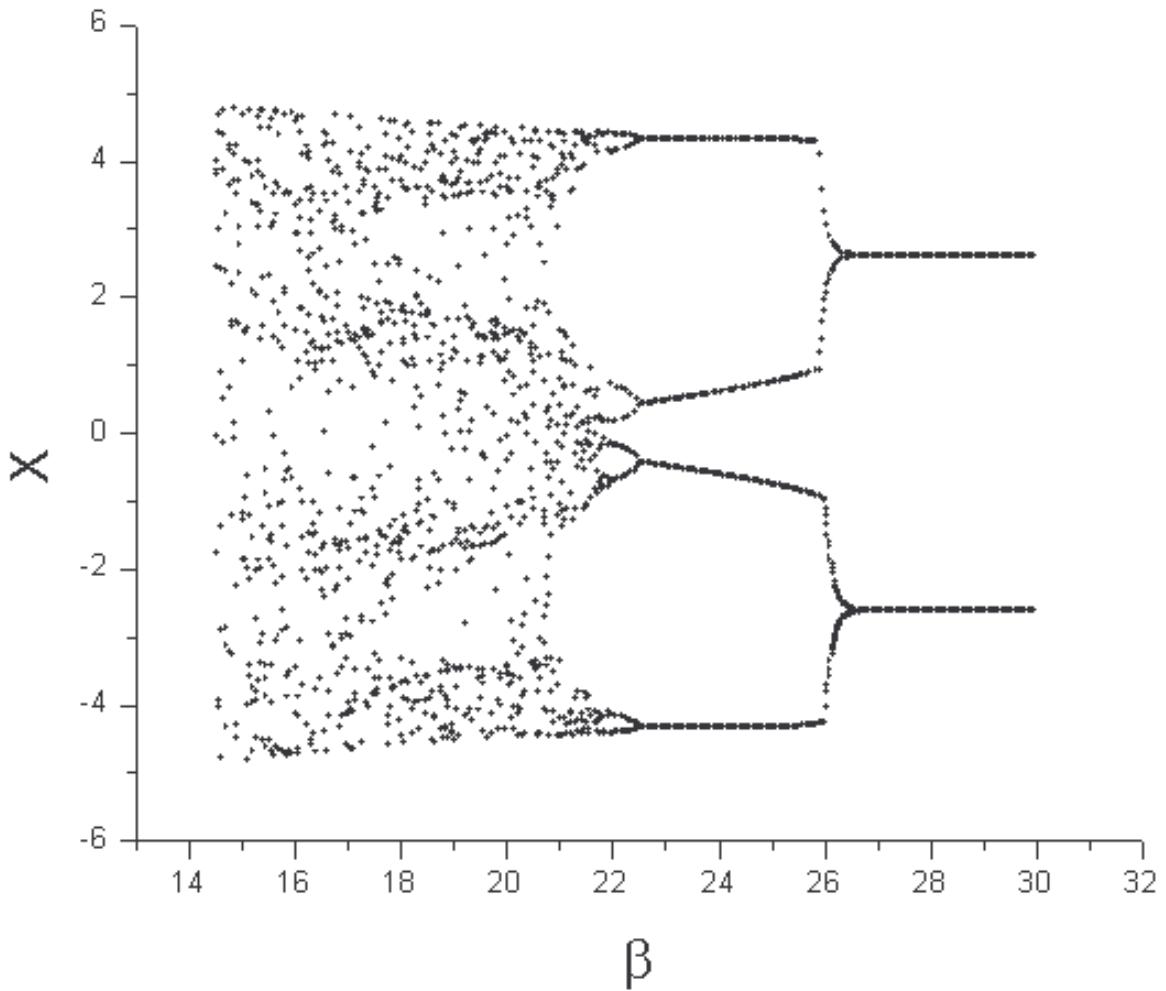
$$\begin{aligned}
 \frac{dx_2}{d\tau} &= \alpha[y_2 - x_2 - f(x_2) - k \sin(2\pi ft)] + \delta(x_1 - x_2) \\
 \frac{dy_2}{d\tau} &= x_2 - y_2 + z_2 \\
 \frac{dz_2}{d\tau} &= -\beta y_2
 \end{aligned}$$

where  $f(x) = bx + \frac{1}{2}(a-b)[|x+1| - |x-1|]$  is a three piece-wise linear function with the two external linear segments of slope  $b$  and the internal segment with slope  $a$ ,  $\delta$  is the coupling constant and reflect the value of the coupling resistance. The parameters  $\alpha$  and  $\beta$  are identical in both circuits.  $\eta$  and  $\kappa$  are parameters and  $N(0, 1)$  is a Gaussian white noise function.  $f$  is the frequency of a the sinusoidal perturbation applied to circuit II. This model is capable to display very dynamic behavior and had been used in several dynamical theory systems [8-10].

A typical value for the parameters of the Chua's circuit had been selected in order to each circuit is in a chaotic state when they are not coupled ( $\delta=0.0$ ) and without any additional perturbation signal [2, 3]:  $\alpha=10.0$ ,  $b=18.0$ ,  $a=-1.40$ ,  $b=-0.70$ . Figure 3 shows a time series for the state variable  $x_1$ ,  $x_2$  for  $\delta=3.0$ ,  $\eta=0.20$ ,  $\kappa=0.09$ , and  $f=0.30$ .



**Figure 3: Time series diagram of the behavior of the two dynamic variables of the system, corresponding to the voltages on each circuit.**

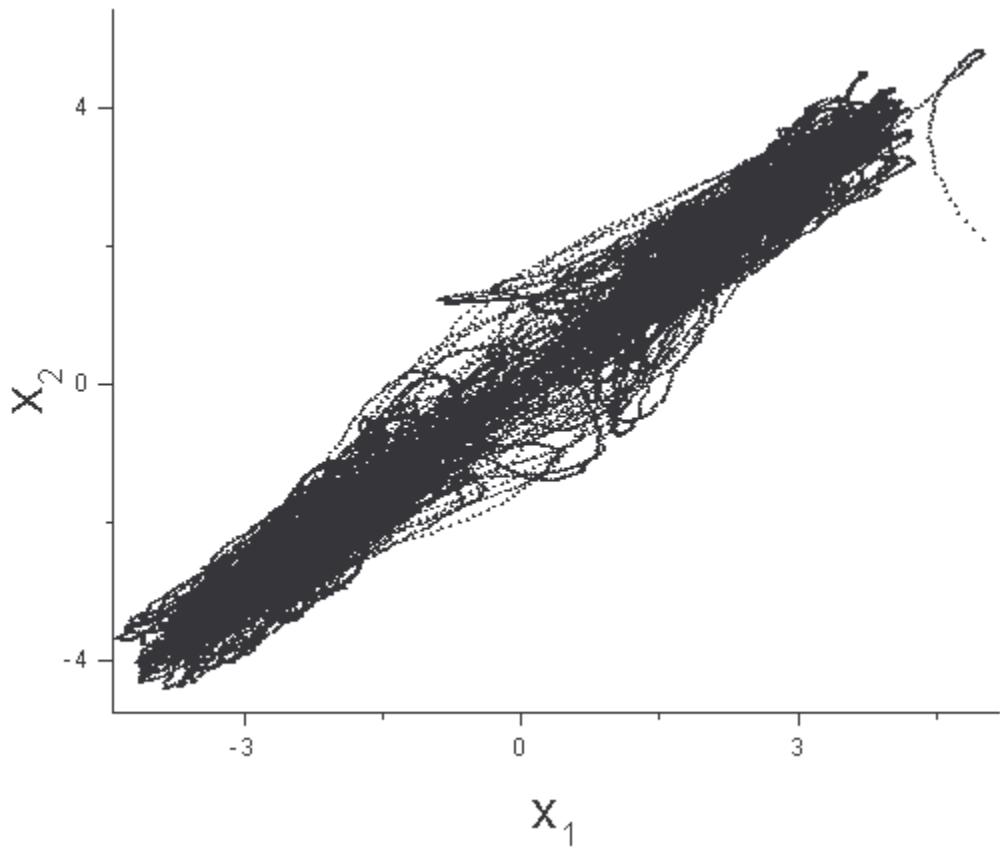


**Figure 4: Bifurcation diagram using  $\beta$  as parameter. Two dynamical behaviors are evident, a chaotic for small values of  $\beta$ , and a periodic behavior for  $\beta>22$ .**

According to figure 4, the dynamics of the system corresponds to an inverse bifurcation of double period, beginning is a state of chaos for low values of  $\beta$ , going through high period states and finishing in a state of period two. Although by increasing the parameter  $\beta$ , we reach a state of period one. Explanation of this behavior is based on the nonlinear-negative impedance that the circuit employs. The range of chaotic behavior belongs to the line in Fig. 2 with the higher absolute slope.

To realize how much the system can synchronize, it has been obtained the correlation diagram shown in Figure 5. Taking into account that to one of the circuits it has been connected a source of noise in series, and to the other circuit a periodic modulation, we can easily conclude that there is a very good correlation coefficient is found to be 0.9986 using the same parameters that for the time series diagram of Figure 3. These last facts show that it is possible to synchronize two Chua's circuits, by applying noise to one of them and a modulated signal to the other.

This chart shows the high degree of correlation between the two main variables. It has been using a log time for calculations to ensure the synchronization, this causes the high density of point around the line  $X_2 = X_1$ .



**Figure 5:** Correlation diagram using both principal variables  $X_1$  and  $X_2$ . The high correlation ( $r = 0.9986$ ) is evident. The high density of point around the line  $X_2 = X_1$  is based on the long time simulation.

#### 4. CONCLUSIONS

The ease of assembly of the Chua oscillator circuit makes it very interesting for study of chaos. The general usefulness of the impedance converter circuit is made manifest, being fundamental in the development of the circuit. The versatility of response of amplifiers in the case of a nonlinear function is manifest in the Chua diode. The theory of nonlinear systems is a key for physical implementation due to the characteristics of the methods and devices used. It is now clear that such a simple system to implement can be used to study the way two systems can be synchronized, even if they are non-identical as in this case where to one of them has been added a noise source, while to the other one a periodic voltage source.

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