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Modelling of Centralized Demand in Supply Chains Using Integer Programming

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ABSTRACT

Collaboration between members of a supply chain has commonly been recognized as a strategy for increasing operational efficiency and reducing costs. In particular, sharing such information as sales information can significantly provide benefits to the supply chain management. These benefits stem from the reduction of uncertainty, allowing for better decision making. Due to the benefits of collaborating along the supply chain, many techniques for achieving collaboration have been proposed, such as Collaborative Planning and Forecasting Replenishment (CPFR). It is also common to share individual forecasts along the supply chain. In particular, many firms centralize customer's demand information making it available to every upstream member of the supply chain. In this paper we, propose to use a mixed integer programming model of a simplistic supply chain, where unobservable customer's demand is forecast using an exponential smoothing model. We compare the benefits of using centralized demand forecast by comparing the optimal costs of the decentralized model versus the costs of the centralized model and identify the differences in performance between the two models. The results further support the benefits of collaborative approach in supply chain management, reported by other authors.

Keywords: Supply Chain, Collaboration, Centralized Demand, Mathematical Programming, Modelling

1 INTRODUCTION

Collaboration between members of a supply chain has commonly been recognized as a strategy for increasing operational efficiency and reducing costs. In particular, sharing such information as: technology, know-how, prices, customer's profiles, data, designs, specifications, order history, and sales forecasts, can significantly provide benefits to the supply chain management. These benefits stem from the reduction of uncertainty, allowing for better decision making. For instance, consider a manufacturer who constantly gets information from the point of sale (POS); he may use this information to improve his aggregate production planning on the basis of information such as, say, the implementation of a new marketing strategy that reduces product's price, temporarily increasing the products demand.

A well studied consequence of information sharing failure through the supply chain is the *bullwhip effect*, which suggests that upstream supply chain members tend to exaggerate the true demand of customers (Baganha and Cohen 1998; Kahn 1986; Metters 1997; Lee et al. 1997). The term *bullwhip effect* was coined at Procter and Gamble to describe the behaviour in the orders between customers and suppliers of Pamper Diapers (Lee et al. 1997). It was soon clear that many firms observed a similar effect in their supply chain (Baljko 1999) and, as a consequence, the study of the bullwhip effect became of interest.

Due to the benefits of collaborating along the supply chain, many techniques for achieving collaboration have been proposed. For instance, Collaborative Planning and Forecasting Replenishment (CPFR) enables a single, mutually owned demand plan to be used by every member of the supply chain (Holmström et al. 2002; Seifert 2003). It is also common to share individual forecasts along the supply chain. In particular, many firms centralize customer's demand information making it available to every upstream member of the supply chain (Lee et al. 1997).

Centralized demand helps reduce operational costs at the supply chain. However, it should be kept in mind that under normal circumstances, using a centralized demand will reduce the bullwhip effect from the supply chain but it will not completely eliminate it (Chen et al. 2000). Furthermore, the reduction in costs between a centralized and a decentralized supply chain increases when moving upstream the chain.

In (Croson and Donohue 2003) an experiment was conducted to compare the effect of centralized demand, further confirming its benefits. In this paper we, differently, propose to use a mixed integer programming model of a simplistic supply chain, where unobservable customer's demand is forecasted using an exponential smoothing model, such as in (Chen et al. 2000). We compare the benefits of using centralized demand forecast by comparing the optimal costs of the decentralized model versus the costs of the centralized model and identify the differences in performance between the two models.

The rest of the paper is organized as follows. In Section 2 we introduce the mixed integer programming model of the supply chain by describing in detail its elements. In Section 3 we describe how centralized demand forecast is implemented in the model. Section 4 describes the Monte Carlo experiment used to study the effect of centralized demand and shows its results. Conclusions are presented in Section 5.

2 A MATHEMATICAL PROGRAMMING MODEL OF SUPPLY CHAIN

In this section we describe the mathematical programming model used to represent a multi-echelon, multi-period planning horizon, single-product, supply chain. Specifically, the supply chain consists of four stages: retailer, wholesaler, distributor, and manufacturer, as shown in Figure 1.

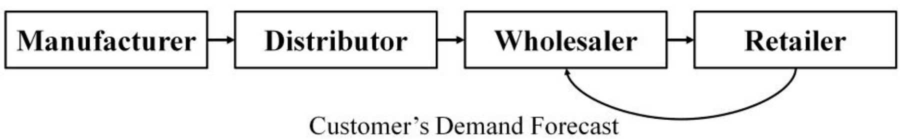


Figure 1: Supply chain representation with customer's demand forecast

The curved arrow in Figure 1 stands for the fact that the customer's demand forecast obtained by the retailer is used in ordering to the wholesaler.

2.1 NOTATION USED

The following is the notation used in this paper.

t	:	Time period in planning horizon, where $t = 1, \dots, T$
s	:	Supply chain stages, where s = 1 (for retailer), 2 (for wholesaler), 3 (for distributor), and 4 (for manufacturer)
I_t^s	:	Inventory level at stage s and at the end of time period t
O_t^s	:	Amount of products ordered by stage s and received by stage $s+1$, at period t. In the formulation of the problem, O_t^s is replaced by $O_t^s = O_t^{s,+} - O_t^{s,-}$
D_t	:	Amount of products demanded by customers at the beginning of time period t
S_t^s	:	Amount of products shipped from stage s to stage $s-1$ during time period t
h^s	:	Holding costs at stage s per unit in inventory at the end of a time period
p^s	:	Backlogging costs at stage s per unfulfilled ordered unit at the end of a time period

c^s	:	Unitary cost of transporting between stage s and stage $s-1$
K^s	:	Cost of placing an order at stage s
o	:	The cost of over offering of supply to the customer
u	:	The cost of under offering of supply to the customer
y_t^s	:	Binary variable that takes the value of 1 if order from station s at period t and 0 if we do not
US_t	:	Under satisfied demand of customer by retail at period t
OS_t	:	Over satisfied demand of customer by retail at period t
α^s	:	Smoothing parameter used to obtain forecasts F_t^s's
I_0^s	:	Initial inventory level at stage s
F_t^s	:	Forecast of amount of requested in order O_{t-1}^s at the beginning of time period t

2.2 DESCRIPTION OF THE MODEL

At each time period t, each stage s sends an order of O_t^s units to its immediately upstream stage. Due to operational lead times, this order is received L_O time periods later. Orders are placed on an order-up-to policy, i.e., at each time period, every stage orders the amount necessary to completely fulfill its downstream demand. This might not be, however, always possible; unfulfilled orders are then backlogged. At each time period, each stage decides how much to order, locally minimizing its operational costs. It should be noted that this strategy, however, does not necessarily optimize globally the operational costs of the supply chain, since optimization is done without collaboration.

On the other hand, at each time period t, each stage s ships a quantity S_t^s of products to its immediate downstream stage $s+1$. Due to operation lead times and transportation time, the products are assumed to be received L_S time periods later. Each stage has a shipping capacity of C^s, which includes production, financial, supply, transportation, and technical limitations.

The inventory level at the end of a time period t at stage s, denoted by I_t^s, changing from period to period, depends on the amount of received units, as well as the amount of shipped units during time period t.

Figure 2 shows the updates of inventory levels I_t^s, orders placed O_t^s, and shipped amounts S_t^s within each time period. Note that, for the sake of clarity of Figure 2, we assumed $L_O = 0$ as well as $L_S = 0$.

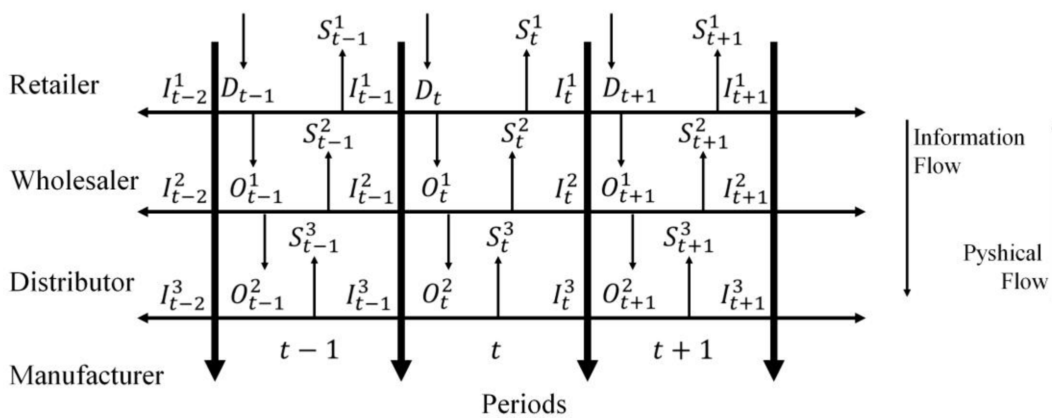


Figure 2: Representation of relations between elements of the model

For every stage s, there is a holding cost h^s for every unit in stock at the end of a time period. Equivalently, there is a penalty cost p^s for unfulfilled backlogged orders at stage s. There is also a fixed cost K^s for placing an order at stage s independently of the number of units ordered. Finally, there is a cost c^s for each unit shipped from stage s to stage $s-1$. These costs combined form the total cost of the supply chain management.

The customer's demand at period t is D^t and is considered to be random. However, since at every stage the exact amount of future demand is not known, we need to predict it as F_t. In Section 2.4 we show the exponential smoothing forecast model used to obtain F_t in this paper.

2.3 FORMULATION OF MATHEMATICAL PROGRAMMING MODEL

The decision variables, objective function, and constraints of the model are detailed in this section.

2.3.1 Decision Variables

The decision variables used in this program are:

$O_t^{s,+} \geq 0$, $O_t^{s,-} \geq 0$, $S_t^s \geq 0$, $I_t^s \geq 0$, $US_t \geq 0$, $OS_t \geq 0$, $y_t^s \in \{0,1\}$.

2.3.2 Objective Function

The objective function we seek to minimize is the total cost of operations of the supply chain, and includes the following costs:

1. ***Cost of holding inventory*** accounts for the cost of having units in inventory at the end of each time period.

$$\sum_{t=0}^{T}\sum_{s=1}^{3} h^s I_t^s$$

2. ***Cost of stocking out*** accounts for the cost of not having units in inventory when demanded and is obtained based on the number of unsatisfied orders at the end of each time period.

$$\sum_{t=0}^{T}\sum_{s=1}^{3} p^s\left(O_t^{s-1,+} - S_t^s\right)$$

3. ***Cost of placing an order*** includes the administrative and overhead costs of placing an order and is independent of the amount ordered.

$$\sum_{t=0}^{T}\sum_{s=1}^{3} c^s O_t^{s,+}$$

4. ***Cost of over and under supply to customer*** is related to the fact that we should not provide more nor less of the customer's needs. It is related to the forecast technique used and to the availability of downstream supply.

$$\sum_{t=0}^{T}(oOS_t + uUS_t)$$

The objective function is then

$$\min z = \sum_{t=0}^{T}\sum_{s=1}^{3} h^s I_t^s + \sum_{t=0}^{T}\sum_{s=1}^{3} p^s\left(O_t^{s-1,+} - S_t^s\right) + \sum_{t=0}^{T}\sum_{s=1}^{3} c^s O_t^{s,+} + \sum_{t=0}^{T}(oOS_t + uUS_t).$$

2.3.3 Constraints

The optimal solution must satisfy the following constraints:

1. ***Inventory level constraint*** updates the amount of available inventory at each stage period to period.
$$I_t^s = I_{t-1}^s + S_{t-L_S}^{s+1} - S_t^s, \qquad \text{for } t = 1, \dots, T \text{ and } s = 1,2,3.$$

2. ***Ordering amount constraint*** defines the amount to order, following an order-up-to policy. If there is enough inventory to satisfy the order received at period t, we will not order during period t.
$$O_t^s = \max(0, O_{t-L_O}^{s-1} - I_{t-1}^s), \qquad \text{for } t = L_O + 1, \dots, T \text{ and } s = 1,2,3.$$

   Note that since we do not know the customer's demand at the beginning of each time period, we replace O_t^1 by F_t for $t = 1, \dots, T$.

   To linearize this restriction we split variable O_t^s into two variables $O_t^{s,+} \geq 0$ and $O_t^{s,-} \geq 0$, as follows
$$O_t^{s,+} - O_t^{s,-} = O_{t-L_O}^{s-1} - I_{t-1}^s, \qquad \text{for } t = L_O + 1, \dots, T \text{ and } s = 1,2,3.$$

   In variable $O_t^{s,+}$ the positive part of the order will be stored. If $O_{t-L_O}^{s-1} < I_{t-1}^s$, then we will have $O_t^{s,+} = 0$, so that we do not need to order during period t.

   To force that either $O_t^{s,+} = 0$ or $O_t^{s,-} = 0$ we use binary variable y_t^s such that
$$O_t^{s,+} \leq y_t^s M, \qquad \text{for } t = 1, \dots, T \text{ and } s = 1,2,3,$$
$$O_t^{s,-} \leq (1 - y_t^s) M, \qquad \text{for } t = 1, \dots, T \text{ and } s = 1,2,3,$$
   where M is a *very large* value.

3. ***Shipping amount constraint*** makes sure that the shipping amount is less than the available inventory.
$$S_t^s = \min(O_{t-L_O}^{s-1,+}, I_{t-1}^s + S_t^{s+1}), \qquad \text{for } t = L_O + 1, \dots, T \text{ and } s = 1,2,3.$$

   To linearize this restriction we separate it into two inequalities
$$S_t^s \leq O_{t-L_O}^{s-1,+}, \qquad \text{for } t = L_O + 1, \dots, T \text{ and } s = 1,2,3,$$
$$S_t^s \leq I_{t-1}^s + S_{t-L_S}^{s+1}, \qquad \text{for } t = L_O + 1, \dots, T \text{ and } s = 1,2,3.$$

   It should be noted that, based on $I_t^s = I_{t-1}^s + S_{t-L_S}^{s+1} - S_t^s$ and the fact that $I_t^s \geq 0$, then $S_t^s \leq I_{t-1}^s + S_{t-L_S}^{s+1}$ is always satisfied.

4. ***Customer's satisfaction constraint*** determines how much of the customers demand is under-satisfied or over-satisfied by the retailers supply.
$$US_t - OS_t = \sum_{i=1}^{t} D_i - I_t^1, \qquad \text{for } t = 1, \dots, T.$$

2.4 DEMAND FORECAST

In our model, we consider that the retailer can only observe the customer's demand at the end of each corresponding time period. However, orders to the wholesaler must be sent at the beginning of each time period in advance, anticipating the demand. Thus the need of a forecast. We propose to use an exponential smoothing model to forecast, as in (Chen et al. 2000).

The formula of the exponential smoothing forecast is:

$$F_t = \sum_{i=0}^{\infty} \alpha (1 - \alpha)^i D_{t-1-i},$$

where α is called the smoothing parameter and is restricted to $0 \leq \alpha \leq 1$. The larger the value of α, the more weight we are giving to the most recent observations.

3 CENTRALIZED DEMAND FORECAST

In collaborative supply chains, forecasts are performed on the basis of each stage's own historical orders plus additional information shared by other stages. Specifically, the customer's demand is usually shared with upstream suppliers through real time communications using technologies such as electronic data inter-change (EDI) located at the POS (Croson and Donohue 2003; Chen et al. 2000). As an example, It was reported in (Hammond 1993) that retail giant Wal-Mart transmits its sales and inventory information in real time from the stores directly to its suppliers via satellite so that suppliers can make better operational decisions based on historical real sales data rather than on educated guesses (Simatupang and Sridharan 2002).

The centralized demand forecast in a Supply Chain is represented in Figure 3.

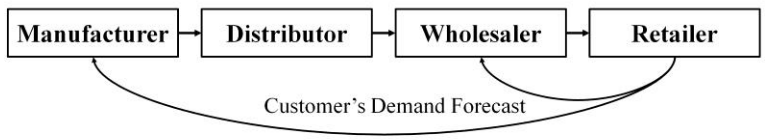


Figure 3: Supply chain representation with centralized customer's demand forecast

To make the customer's demand directly available to the manufacturer, we substitute the ***Ordering amount constraint*** $O_t^{3,+} - O_t^{3,-} = O_{t-L_O}^2 - I_{t-1}^3$ with

$$O_t^{3,+} - O_t^{3,-} = \beta O_{t-L_O}^2 + (1-\beta)F_t - I_{t-1}^s \qquad \text{for } t = L_O + 1, \dots, T,$$

where $0 \leq \beta \leq 1$ is a factor that describes the weight that the manufacturer provides to the wholesaler's order and to the forecast of the customer's demand. Variable β becomes another decision variable in the new model. The rest of the model is kept the same.

In the next section we show results comparing the performance of forecasting at each stage with the customer's demand information and without. Furthermore, we show how the operational costs of the supply chain goes down when the demand information is centralized.

4 MONTE CARLO SIMULATION

In this section we describe the simulation used to compare the non-centralized demand model with the centralized demand model. We implement the models using solver CPLEX in GAMS version 23.9.1. We consider a 48-week planning horizon and use inventory costs of $h^s = 1$, out of stock costs of $p^s = 5$, ordering costs of $K^s = 2$, and unitary costs of $c^s = 10$, for every stage s.

The demand seen by the retailer will be described, as done by other authors (e.g., Lee et al. 1997; Kahn 1986; Chen et al. 2000), by a stochastic process $\{D_t\}_{t \geq 0}$ of the form

$$D_t = \mu + \rho D_{t-1} + \epsilon_t \qquad \text{for } t = 2,3, \dots T,$$

where μ is a nonnegative constant, ρ is a correlation factor such that $|\rho| < 1$ and ϵ_t is a symmetric random variable centered at 0 and with finite variance σ^2. It was further shown in (Chen et al. 2000), that

$$E[D_t] = \frac{\mu}{(1-\rho)} \text{ and } V(D_t) = \frac{\sigma^2}{(1-\rho^2)}.$$

We use the values $D_1 = 200$, $\mu = 200$ and $\rho = 0.4$, and consider ϵ_t to be uniformly distributed between 50 and 150 units. The random generated demand used in the simulations is shown in Figure 4. Additionally, Figure 4 also shows the forecast obtained using exponential smoothing with an optimal $\alpha^* = 1$ (when using $\alpha = 1$, the exponential smoothing forecast is usually called the naïve forecast).

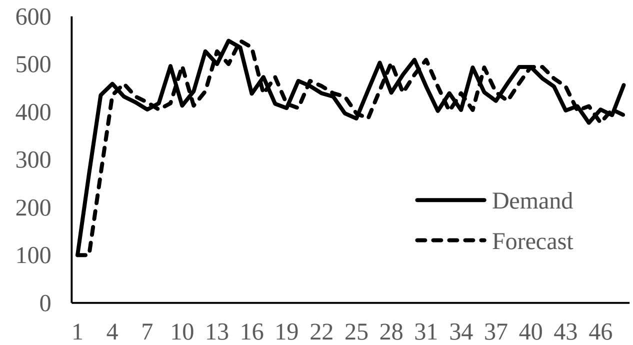
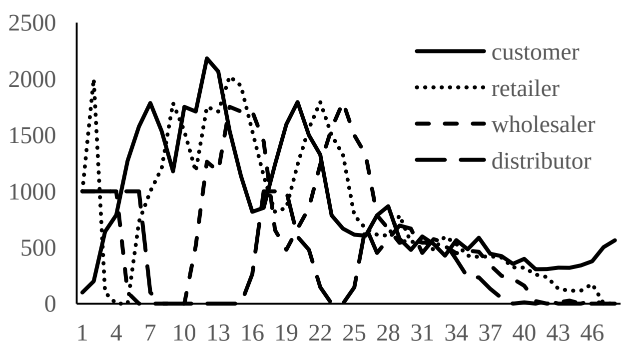
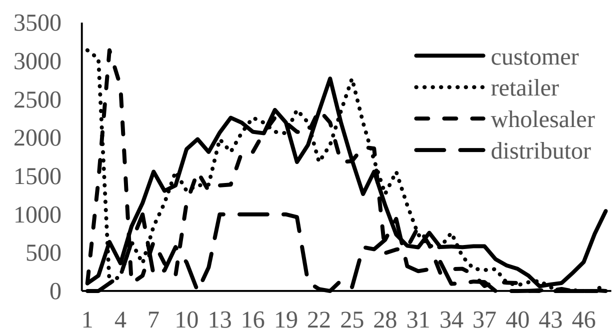


Figure 4: Generated demand and exponential smoothing forecast

Figure 5 (a) shows the amount ordered by each stage of the supply chain in their optimal decision. The optimal cost of managing the whole supply chain during the complete planning horizon was $ 1,759,427.



(a)



(b)

Figure 5: Amount ordered at each stage: (a) decentralized demand, and (b) centralized demand

By contrast, in Figure 5 (b) we show the amount ordered by each stage of the supply chain under a centralized demand forecast model. Note that the variability of the amount ordered by the distributor to the manufacturer has been significantly reduced. At the same time, the optimal cost of managing the whole supply chain during the complete planning horizon was $ 1,571,241.

5 CONCLUDING REMARKS

In this paper we study the effect of centralized demand in the performance and operational costs of a four-echelon multi-period supply chain. We used a mixed integer programming model of the supply chain and incorporated an exponential smoothing model to forecast the demand at the retailer. To study the effect of centralized demand we allow the distributor to have access to customer's demand and incorporate this information in the orders placed and sent to the manufacturer. The results further support the benefits of collaborative approach in supply chain management reported by other authors. Furthermore, it was shown that, in particular, the variability in the distributors orders is significantly reduced due to centralizing the demand.

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