*Twelfth LACCEI Latin American and Caribbean Conference for Engineering and Technology (LACCEI'2014)*
*"Excellence in Engineering To Enhance a Country's Productivity" July 22 - 24, 2014 Guayaquil, Ecuador.*

# Effect of Lumped Mass on the Fundamental Frequency of a Riveted Plate

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## ABSTRACT

During the service life of an aircraft, it may experience various form of structural damage. These may include damage due to the weather such as a hail storm, human negligence such as mishandling of equipment including deicers during the winter or dropping a tools onto the wing surface.

As a result of damage, external surfaces of an aircraft may need to be repaired since replacing an entire panel may not be very economical. The most common form of repair is usually patching the skin of the aircraft. One issue with patching is that the patch now acts as a lumped mass and can affect the behavior of the panel onto which it is mounted.

In this paper, the effect of lumped mass on the fundamental frequency of a panel located on the aircraft wing was studied using finite element analysis (FEA). To achieve this, a thin curved plate with riveted ends will be modeled using a CAD software. A patch (lumped mass) was then riveted onto this plate, while the location of the patch on the plate varied. The effect of lumped mass on fundamental frequency is presented, also the effect of patch location vs. frequency is presented.

**Keywords:** riveted plates, curved plates and natural frequency

## 1. Introduction

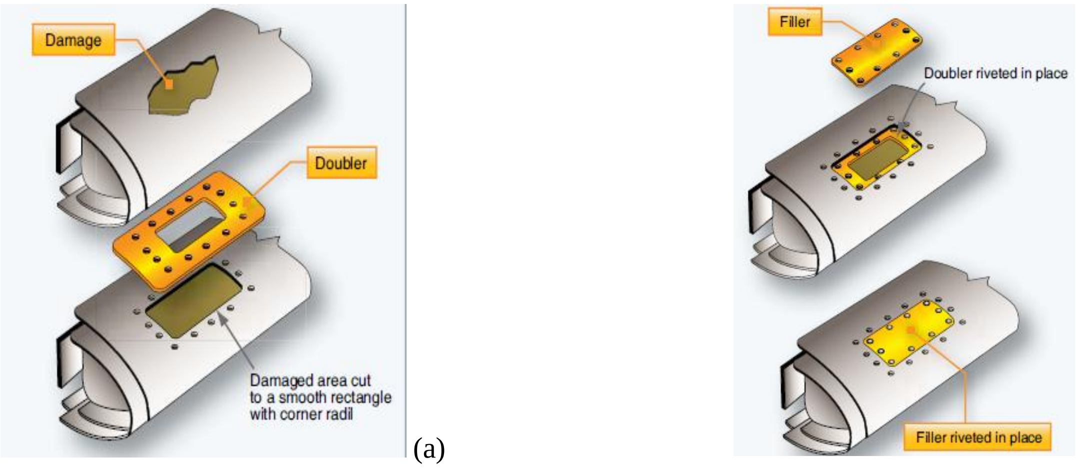
Every object has an inherent natural frequency, this natural frequency changes due to changing the shape, size and mass of the object. When external excitation or excited frequency is equal to the natural frequency of the object, Resonance occurs.

Considering an aircraft wing during flight, the air passing over the wings creates vibration on the skin of the wing which is caused by drag. Other sources contribute to the vibration as well which include the engine vibrations, turbulence and wing loading. This phenomenon has to be kept in mind while designing the aircraft that the drag of the air and other external forces does not create a frequency equaled to that of the aircraft wing, if this is not followed, the wing can collapse easily causing severe damage.

This paper aims at studying the change in frequency of a wing panel of an aircraft after it is repaired (patched) from damages during the service life of the aircraft. Damages can be due to bird strikes, heavy hail, tools or luggage mishandling as shown in Fig.1.

**Figure 1:** Aircraft skin damage due to bird strikes [1]

During the repair of a damaged area, the panel is cut as shown in Fig 2. and a filler and doubler are riveted to repair the damaged area. The rivets follow a specific pattern so that the aerodynamic properties of the wing are not affected. The holes for placing the rivets are drilled onto the patch then to the wing panel itself (around the damaged area) and finally the rivets are applied to fix the parts together.



(a) (b)

**Figure 2(a) & 2(b):** Application of Patch on damaged area [2]

## 2. Description of Proposed Solution

For this study an aluminum panel of dimensions 24-in by 36-in with a thickness of .25-in was used. The panel along with rivets and rivet holes were then modeled using a CAD software. Care was taken to replicate the actual dimensions and location of rivet and rivet holes. The calculation for rivet location was carried out using equation (1). [3]

$$(1\frac{1}{2})D + G = L \qquad (1)$$

Where: D = the rivet diameter, G = the grip, total thickness of material, L = the total length of the rivet.

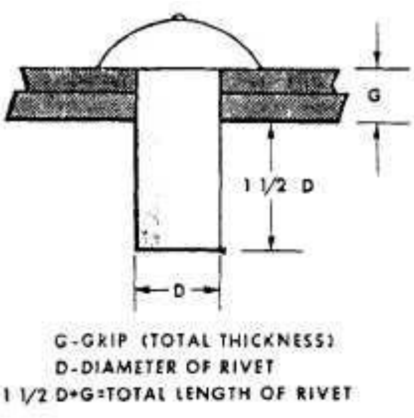


**Figure 3:** Specification for rivets

The analytical solution for the fundamental frequency was then calculated. A finite element analysis of the plate was then conducted with proper mesh refinement to obtain a reliable model. This model was them modified by adding a patch to it at various locations over its surface as shown in figure 4. The doubler was 7.5 inches long and 4.5 inches wide. The filler was 6.25 inches long by 3.25 inches wide. A numerical analysis was then carried out to study the ***effect of the patch location on the fundamental frequency of the plate***.

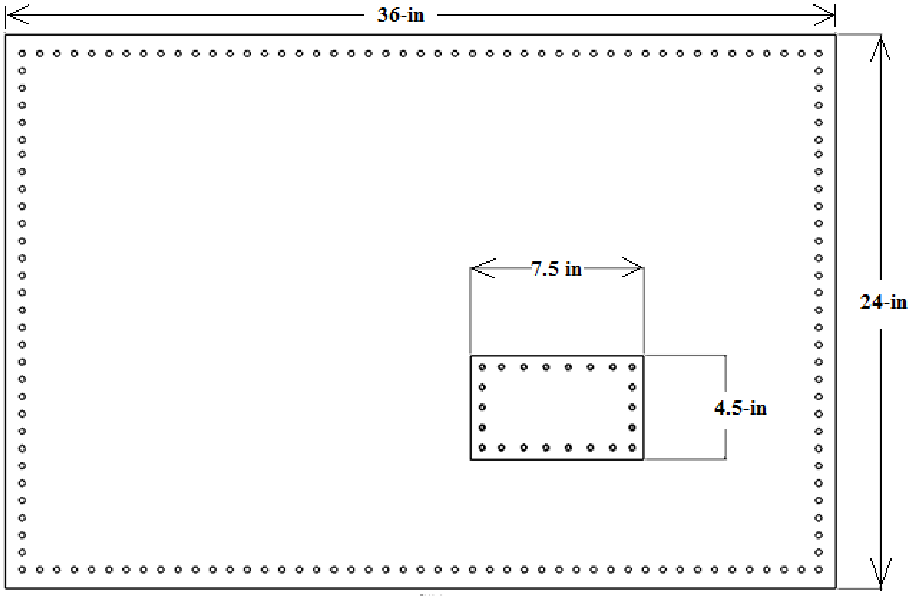


**Figure 4:** Schematic of panel and patch

In addition a curved plate was then modeled and a numerical study was carried out focusing on the ***effect of curvature and patch location on the fundamental frequency of the plate***. A schematic of the curved plate is shown in figure 5 below where the angle θ was varied. Finally a study was carried out to study the effect of the having different size patch on a wing panel. This was done by selecting a specific location on the plate, add a patch and vary its mass. An equation was then developed to study the ***effect of mass on the fundamental frequency of a plate***.

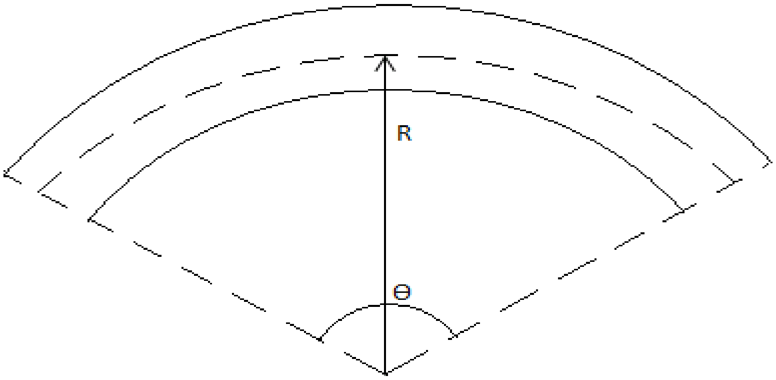


**Figure 4:** Schematic of curved plate

## 3. Application

Since aircraft repair is most needed for all aircrafts in service, this analysis will provide a better understanding for aircraft wing repair. This would save time and money for the repair because it will be known whether to replace a whole wing panel or just to apply a patch to the damaged area with its known change in its natural frequency. This would also help in future wing design and repair concepts because of the curved patch natural frequency analysis. Apart from an aircraft panel repair, this research work could also be applied to any machinery or vehicles which rely on repairs and operate in high vibrational environments such as spaceships or aircraft carriers etc.

## 4. Theoretical Analysis

The main panel is 36 inches long and 24 inches wide with a thickness of .25 Therefore,

a = 36in = .9144m, b = 24in = .6096m, 2h = 0.25in ∴ h = $3.175^{-3}$m

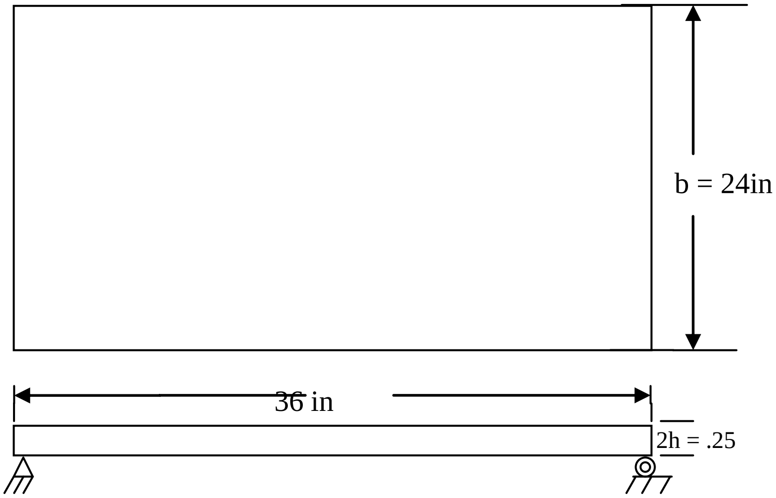


**Figure 5:** Schematic of panel

The analytical solution to the natural frequency of a simply supported plate [4] which is given in equation 2

$$\omega_{mn} = \sqrt{\frac{D\pi^4}{2\rho h}}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) \qquad (2)$$

Where

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \quad \textit{Bending Rigidity}$$

$$E = \textit{Elastic Modulus}$$

$$\rho = \textit{Density}$$

$$\nu = \textit{Poisson's ratio}$$

Using Values for aluminum the bending rigidity was found to be

$$E = 2x10^{11}\,\frac{N}{m^2}$$

$$\rho = 7860\,\frac{kg}{m^3}$$

$$\nu = 0.266$$

$$D = \frac{2\times10^{11}\times\left(3.175\times10^{-3}\right)^3}{12\left(1-0.226^2\right)} \qquad \rightarrow D = 562.1452Nm$$

Letting $m$=$n$=1, the fundamental frequency is given by:

$$\omega_{11} = \left(1.1960 + 2.6909\right)\sqrt{\frac{562.1452\times 3.14^4}{2\times 7860\times\left(3.175\times 10^{-3}\right)}}$$

$$\underline{\overline{\omega_{11} = 128.7447}}Hz$$

Conversion of Fundamental Frequency to Frequency Coefficient: Conversion is made to frequency coefficient so that eliminate the dependency on material properties. [5]

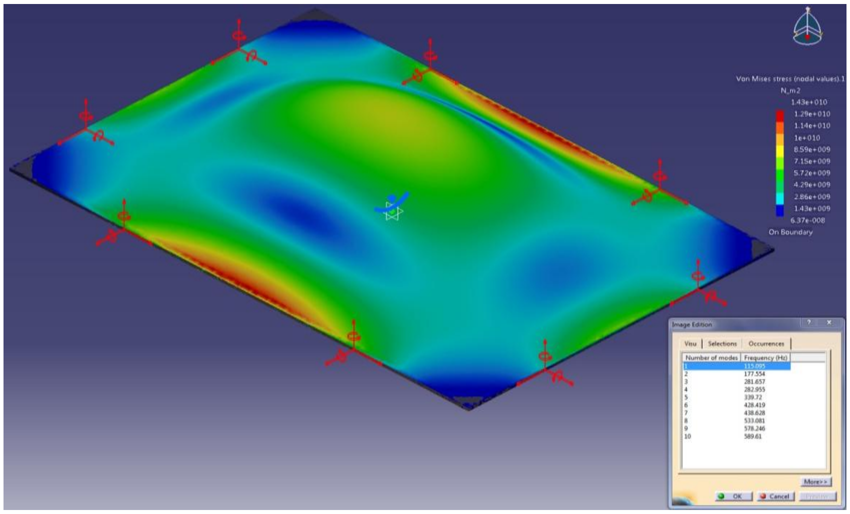
$$\lambda_{11} = \omega_{11} a\times b\sqrt{\frac{\rho}{D}}$$

$$\lambda_{11} = (128.7447\times0.9144\times0.6096)(\sqrt{\frac{7860}{562.1452}}) = 3.739271672 \qquad (3)$$

$$\lambda_{11} = 268.3475$$

### 5.1 Numerical Analysis

A simple supported aluminum plate was modeled In a CAD software using the previously mentioned dimensions and material properties used in the analytical analysis and are shown in Fig 6 below. A mesh convergence study was conducted and results show in figure 7.



| Number of modes | Frequency (Hz) |
|---|---|
| 1 | 115.095 |
| 2 | 177.554 |
| 3 | 281.657 |
| 4 | 282.955 |
| 5 | 339.72 |
| 6 | 428.419 |
| 7 | 438.628 |
| 8 | 533.081 |
| 9 | 578.246 |
| 10 | 589.61 |

**Figure 6:** Model of a simply supported plate

### 5.2 Mesh Convergence Study

It can be seen from the plot that the mesh size has a great impact on the numerical results, but as the mesh size approach 0.2, the effect becomes minimal. Therefore in a real world application, using the mesh size of 0.2 or 0.15 will not have a large effect on the numerical results, however it was found that the computational time increased by more than 100% . Since a large computational time is not an issue in this project.

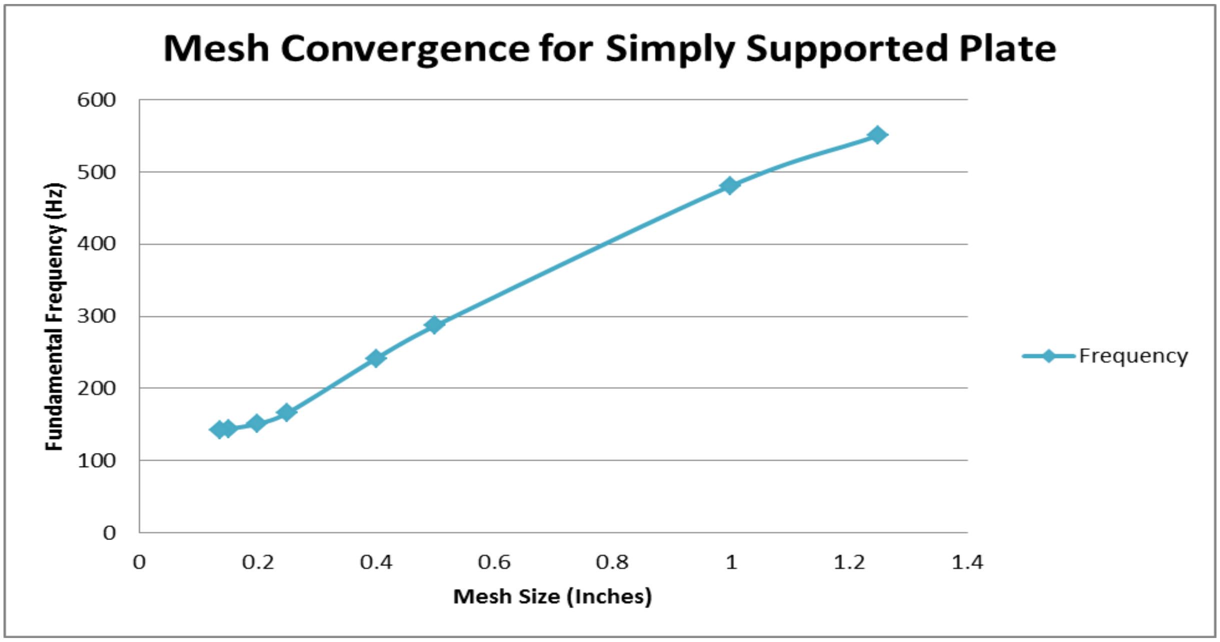


**Figure 7:** Mesh Convergence Study

The mesh size used in this paper was 0.2 since it gives the best results in comparison to the analytical solution. The comparison is given in table 1. Based on the results given in table 1, it will be assumed that CATIA provides an acceptable approximation of the natural frequency of simply supported plate. Based on this assumption, it will further be assumed that making the plate more complex by changing the boundary conditions from simply supported to riveted, and also bending the plate, the numerical results obtained will be acceptable

**Table 1:** natural frequency of a simple supported rectangular plate

| Theoretical | Numerical |
|---|---|
| 115.095 Hz | 128.744 Hz |

## 5. 3 Effect of Patch Location

To study the effect of patch location on the fundamental frequency, a patch was made with a mass ratio of 0.04 or 4 % and added to the plate as shown in figure 8. For obtaining the frequency of the patched plate, the patch was placed at 2- in increment along the width (x-direction) and 2-in increment along the length (y-direction) of the plate.

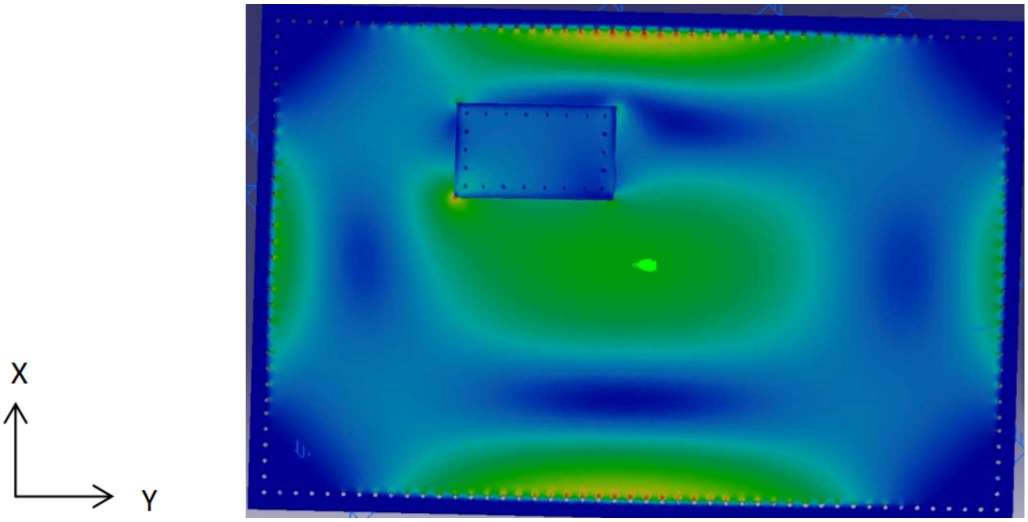


**Figure 8:** Model of plate and patch

The fundamental frequencies along the length and width of the plate were obtained using the finite element method. Since a 3-D graph is required to represent the frequency of the patch over the plate, a 2-D simplification is presented here with explanation. In Figure 9, the center of the patch was placed 3-in from the edge of the x-direction, and varied along the y-direction; the same steps were repeated where the patch was placed 4-in away from the edge of the y-direction and varied along the x direction. The results were then plotted in figure 9, where the origin is assumed to the center of the plate. The fundamental frequency of a plate without any lumped mass (patch) is also plotted for comparison purposes. It can be seen that the fundamental frequency along the length of the plate is lower than those along the width of the plate; this corresponds to the plate being stiffer along the width. Considering the frequency along the width of the plate, it can also be seen that the variation of frequency of the patch plate was minimal when compared to the un-patched plate. For both cases however, it can be seen that as the patch approaches the center of the plate, the overall stiffness decreases.

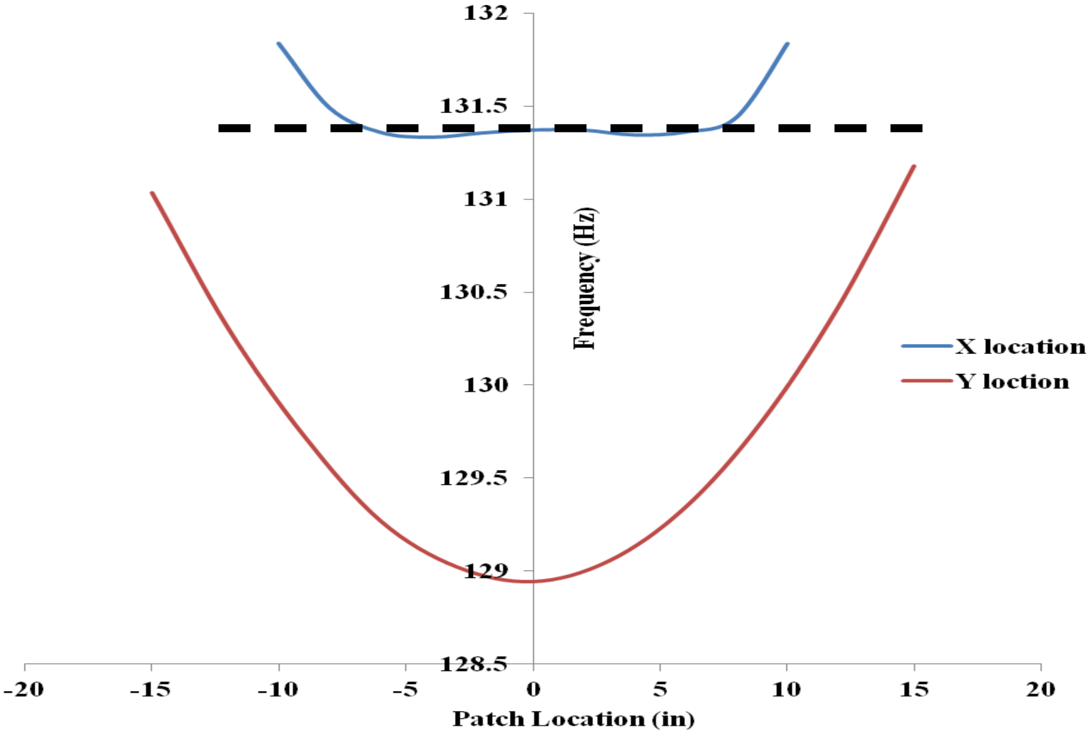


**Figure 9:** Frequency vs. patch location

One very important observation from fig. 8 is the stress field around the patch and the edges of the plate, it can be seen that along the edges, the rivets around the center of the edges will tend to experience a higher stress values. An optimum design would therefore be to maybe use stronger rivets in these locations or increase the numbers of rivets here. It can also be seen that around the edges of the patch, there is some stress concentrations, therefore fillets could be used here to reduce the stress.

### 5.4 Effect of Mass Ratio

After studying the effect of match location and its effect on the fundamental frequency of a 'riveted-end' plate, it was seen that the frequency variation was very small. The attention was then turned to study the effect the mass of the patch would have since the initial patch was only 4% of the plate's mass. For this study the patch of same dimensions were placed at the center of the plate while it's' mass varied. Instead of presenting the frequency values, the frequency coefficient (λ) was presented. The reason for this is to remove the effect of material on the results. The frequency coefficient was calculated based on eq. (3). The results are presented in figure 10. It can be observed that there is a linear relation to the mass and frequency coefficient, hence the frequency. From the best fit line an expression: $\lambda = 236.87m^2 - 205.78m + 126.59$ (where m = mass ratio) was generated for the calculation of the frequency coefficient. This formula can now be used to find the frequency coefficient for a given mass ratio of patch to plate. Using eq. (3) with necessary material properties, the frequency coefficient can then be converted to frequency for that plate.

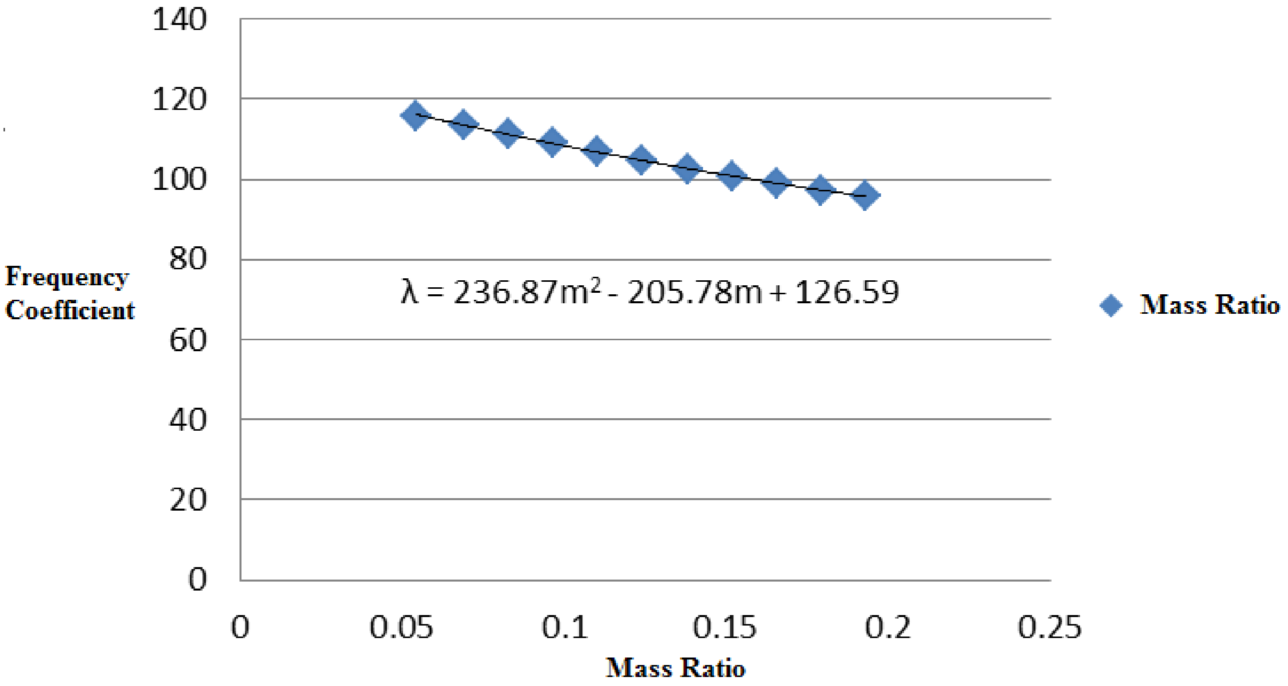


**Figure 10:** Frequency vs. mass ratio

## 6. Conclusion

This report focused on the effect of lumped mass on the frequency of a flat plate. The lumped mass was assumed to be a patch which is used to repair an aircraft panel after damage. It was seen that shorter the panel, the stiffer it would be, hence higher fundamental frequency. If was also found that as the patch lowered the frequency of a flat plate as compared to a plate without patch. As the patch is moved towards the outer edge of the plate, the frequency becomes larger that a plate without any patch. It was also found that the mass ratio of patch to plate increased the fundamental frequency linearly. A second part of this research/future work would be to curve the plate. This would be more applicable to an aircraft skin panel since most of the outer panel on an aircraft structure has some curvature to it.

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