Understanding Dynamics of Combined Cycles in Energy Plants with Theories of Variables Interaction: A pedagogical Approach to the Undergraduate Level of Engineering *

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Abstract— In this note we present a pedagogical approach of variable interactions applied to a 2x2 MIMO system and its confrontation to acquired data from a combined cycle power plant, is studied. The Laplace space is used as a scenario to compute basic operations and subsequently the resulting relations are transformed in convolution integrals. The term responsible of interaction is then simulated and confronted to acquired data from dynamics variable of the power plant. Concretely, attention is paid to the drum level which turns out to be the variable of most importance inside the combined cycle power plant. Our approach is as a pedagogical introduction with a theoretical model sustained even for those cases where the temporal evolution of the drum level is notably affected by intrinsic fluctuations. Thus, a discrepancy between model and data results to be of order of 5%. It would suggest that the interaction variable formalism might be of interest to design model-based control theories which could be applied to those MIMO systems which are plagued of intrinsic and stochastic fluctuations.

I.1 INTRODUCTION

It is well-known the notable role that the advanced mathematical methodologies are playing inside the scenarios of control theory in where the necessity for implementing and optimizing sophisticated control systems is nowadays much more evident because the persistent industrial challenges. Essentially this necessity in implementing robust mathematical schemes into advanced control systems comes from the fact that the set point should be carefully monitored along the control horizon and be capable to anticipate and defeat those possible events containing fluctuations and instabilities system. In particular in power plants as those consisted in combined cycles [1]. A point of importance that should be stressed is that of the presence of instabilities as consequence of the interrelation of dynamic variables during the temporal evolution of the system and their associated random events. It is believed that variable interactions take place on multivariable systems and thus, is a potential source of instability [2]. In effect, as it is observed there the presence of variable interaction would lead to unnecessary stops in plants. Furthermore, it has been proposed that such interaction has a stochastic character in the sense that it cannot be known with precision nor the whole mathematical interaction term can be measured when the system is running. Thus, naives and intuitive models are built. In this note, attention is paid to the possible scenarios where variable interactions take place and how it enters to the phenomenology of the problem. The formalism is expressed in the Laplace space which is commonly a recurrent territory to explore system properties. Once the interaction is fixed then the Laplace quantities are passed to the time through the well-known Faltung theorem. Then the system interaction term is expressed as convolution integral by allowing perform computational tests and simulations to validate the formalism. In order to illustrate all these ideas, the resulting computational simulations are confronted to acquired data from a combined cycle plant aimed to generate about 150 MW of power energy to a substantial portion of Lima city [2]. Mainly in this type of facilities the variable of central importance is that of the drum level which must be ranging in a small window of order of 1% of its nominal value in order to guarantee the stabilize of the engine dynamics appropriately. This note is structured as follows: in second section a brief introduction to the main pieces of the variable interaction formalism is presented. The discussion is based in the Laplace space entirely. In the end of the section, a convolution integral is proposed which involves the interaction term and which subsequently is used to simulate the drum level temporal evolution curve. In third section, an overview on the combined cycle plant together to the variables specification is presented. We provide a plot where the plant variables are displayed. In fourth section, a computational approach for testing the mathematical relations is provided. Finally, in fifth section some conclusions regarding the results of this note are drawn.

I.2. THEORETICAL BACKGROUND

A very useful technique to get information about stability of complex systems is that of using the Laplace space which enables us to go through their crucial properties. It is perhaps well exemplified in the typical operations for extracting the system poles. Among a plethora of cases, the Laplace space is a recurrent territory to explore system robustness and their weakness. A well-known theorem also called the Faltung theorem provides a vision to manage the so-called convolution integrals in where the system acquire some kind of complexities because the presence of the time evolution of their variables.

In this note we stress a pedagogical application of the
Faltung theorem to apply to the cases where a system acquires a different dynamics as consequence of the interaction among their variables. Some studies were performed at the past for the case of a ball mill grinding circuit where was found that the 3X3 MIMO system [3] contains an interesting phenomenology due to the confluence of variables which are described as interactions.

In fact, it is not so well studied in the literature that the grade of importance of the variables of interaction has onto the output ones. We stress the fact that the Laplace space and the Faltung theorem can be aspects denoting the same problem.

This note is organized as follows: in second section in a general way the Faltung theorem is described. In third section a general application to 2X2 MIMO system is provided. It is emphasized the cases with and without interactions. In fourth section, the apparition of a term similar to the Volterra series is remarked [3][4]. In fifth section, a simple exercise with the assistance of MAPLESOFT is performed in order to illustrate the contribution of the interaction in the full output. Finally, some conclusions regarding the results of this short note are drawn.

I.3. OVERVIEW TO THE FALTUNG THEOREM

In many books the Faltung theorem is demonstrated from the multiplication of a two functions in the Laplace space, as follows

\[
\begin{align*}
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_{sx} a_{sy} F(s) F(e^{-st}) e^{-st} dxdy &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_{sx} dyy F(s) F(e^{-st}) e^{-st} dxdy
\end{align*}
\]

(1)

\[
\begin{align*}
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_{sx} dyy F(s) F(e^{-st}) e^{-st} dxdy &= \int_{-\infty}^{\infty} dyy F(s) F(e^{-st}) e^{-st} \int_{-\infty}^{\infty} a_{sx} dxdy
\end{align*}
\]

(2)

\[
\begin{align*}
G(t) &= \int_{-\infty}^{\infty} F_1(t-z)F_2(z)dz
\end{align*}
\]

(3)

Therefore, the Faltung theorem resumes: “the Laplace transformed of \( G(t) \) is also known as the Laplace transformed of the convolution integral o function called \( G(t) \). In fact, this new function is the well-known convolution integral where the argument of only a function is displaced,

\[
\begin{align*}
G(t) &= \int_{-\infty}^{\infty} F_1(t-z)F_2(z)dz
\end{align*}
\]

Integral (4) has actually many applications. However we restrict ourselves to manage it inside the control theory and identification theory. In particular to those containing several inputs/outputs as normally happens in complex processes. Therefore, we describe in a brief manner how to apply the integral (4) for obtaining information of the system.

I.4. APPLICATION TO MIMO SYSTEMS

View of a MIMO System Without Interactions

A 2X2 MIMO system can be described in the Laplace space through a matrix operation which relates in a clearly way the input and outputs as given below

\[
\begin{align*}
y_1(s) &= g_{11}(s) x_1(s) + g_{12}(s) x_2(s) \\
y_2(s) &= g_{21}(s) x_1(s) + g_{22}(s) x_2(s)
\end{align*}
\]

(5)

so the \( g_{ij}(s) \) the transfer functions enclose the system properties. The outputs are then written as follows

\[
\begin{align*}
y_1(s) &= x_1(s)g_{11}(s) + x_1(s)g_{12}(s) \\
y_2(s) &= x_1(s)g_{21}(s) + x_2(s)g_{22}(s)
\end{align*}
\]

(6)

In other words, for a 2X2 MIMO system the outputs are actually derived from the Faltung theorem, in the sense that the products \( x_1(s)g_{ij}(s) \)

\[
\begin{align*}
y_1(s) &= \int_{0}^{\infty} e^{-st} \int_{0}^{t} g_{11}(t-z)x_1(z)dzdt \\
&+ \int_{0}^{\infty} e^{-st} \int_{0}^{t} g_{12}(t-z)x_2(z)dzdt
\end{align*}
\]

(7)

\[
\begin{align*}
y_2(s) &= \int_{0}^{\infty} e^{-st} \int_{0}^{t} g_{21}(t-z)x_1(z)dzdt \\
&+ \int_{0}^{\infty} e^{-st} \int_{0}^{t} g_{22}(t-z)x_2(z)dzdt.
\end{align*}
\]

(8)

Note that this representation is lineal respect to the input and transfer functions.

I.5 View of a MIMO System With Interactions

Our proposal is as follows: for a system under variable interaction, we added a term “by hand” in order to couple inputs as a multiplication. The matrix view is given as

\[
\begin{align*}
y_1(s) &= g_{11}(s) x_1(s) + g_{12}(s) x_2(s) \\
y_2(s) &= g_{21}(s) x_1(s) + g_{22}(s) x_2(s)
\end{align*}
\]

(9)

and the output can be written with an extra term which gives account of the interaction. Moreover, a coupling constant is assumed “\( \lambda \)”

\[
\begin{align*}
y_1(s) &= x_1(s)g_{11}(s) + x_1(s)g_{12}(s) \\
&+ \lambda x_1(s)g_{11}(s)x_2(s)g_{12}(s)
\end{align*}
\]

(10)

Again, the Faltung term is recognized now as the product of 4 terms in the Laplace space. It is the extended version of Eq. (1).

I.6. THE INTERACTION TERM AS A TRUNCATED VOLterra SERIES

We rewrite again the interaction term in the Laplace space

\[
\begin{align*}
x_1(s)g_{11}(s)x_2(s)g_{12}(s)
\end{align*}
\]

(11)
By following the Eq. (4) it is possible to convert (11) to the time with the inverse Laplace transformed:

\[ \int_0^{t_B} \int_0^{t_A} g_{11}(t-z) x_1(z) g_{12}(t-z') x_2(z') dz dz'. \]  

(12)

One can see that the Eq. (12) can be rewritten as

\[ \int_0^{t_B} \int_0^{t_A} h(t-z, t-z') x_1(z) x_2(z') dz dz'. \]  

(13)

By changing the variables: \( t - z = u \), and \( t - z' = v \) (13) gets a different form

\[ \int_0^{t_B} \int_0^{t_A} h(u, v) x_1(t-u) x_2(t-v) du dv. \]  

(14)

This is a second order truncated Volterra series. It is noteworthy that the Volterra series appears as consequence of the variable interaction, solely.

1.7. Computational Test of the Effect of the Interaction

For instance, as illustration of interaction variable we write

\[ y_1(t) = y_{NF}(t) + \int_0^{t_B} \int_0^{t_A} h(u, v) x_1(t-u) x_2(t-v) du dv. \]  

(15)

Where the interaction is assumed to be a second order truncated Volterra series. Aside, it should be noted that the upper integral limits \( t_A \) and \( t_B \) are defined in accordance to the phenomenology of a particular system. In figures 1 is shown the curves of contribution to the interaction terms (or Volterra-like terms) to the output (10). To this end, we have used hyperbolic tangent function as entries (input) and the transfer functions has to be modelled as Gaussians profile. In Fig. (1) is noted the amplitude of the resulting interaction term when the transfer function is a polynomial function of up to 5th order. The tangent hyperbolic is modelled also as a step function \( \frac{1}{1+\exp(x)} \). This is quite approximated to the ones from the Tanh(x). In both cases (see below) there is contribution as seen in the amplitude of curves as a pure effect of variables interaction.

![Fig. 1. Morphology of the output by effect of the interaction for the case where Eq. 10 is used together with a polynomial.](image)

II BRIEF OVERVIEW TO THE INTERACTION MECHANISM IN MIMO SYSTEMS IN THE LAPLACE MECHANISM

A. System Without Interactions: The Common Case

In the language of I/O it is possible to write down the master equation in the Laplace space as follows

\[ Y = H X. \]  

(16)

Where both \( Y \) and \( X \) denote the output and input ones respectively, whereas \( H \) the transfer matrix. Eq. (1) becomes the simplest representation to establish relations between I/O variables. Often (16) denotes a linear system. Let us now to restring ourselves to this debate to a simple system consisting in one of type 2×2 MIMO. For instance, the master equation (1) can be explicitly represented in the Laplace space by

\[ \begin{bmatrix} N(s) \\ P(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} V(s) \\ C(s) \end{bmatrix}, \]  

(17)

where the output ones are related with their input ones through the matrix elements of the transfer matrix. For example the variable \( N(s) \) can be written as a linear combination of their input ones thereby giving

\[ N(s) = G_{11}(s)V(s) + G_{12}(s)C(s). \]  

(18)

This is rather acceptable from the point-of-view of the linear system theory. The way how (18) is written is sometimes called the representation of output \( N(s) \) without interactions.

B. Introduction to Variables Interaction

Let us to suppose that for some physical reason the output variable \( P(s) \) is again inside the processing cycle and therefore gets interaction with \( V(s) \). It implies to modify the equation (18) with an extra term,

\[ N(s) = G_{11}(s)V(s) + G_{12}(s)C(s) + G_{11}(s)V(s)P(s) \]  

(19)

so that in (4) the interaction appears as a product of the input \( V(s) \) by the output \( P(s) \). One can note that the transfer function \( G_{11}(s) \) is kept together to \( V(s) \). It means that no new transfer function is established despite of the fact of the interaction is assumed. By putting the corresponding term of \( P(s) \) as seen in (1) inside (4), one gets

\[ N(s) = G_{11}(s)V(s) + G_{12}(s)C(s) + G_{11}(s)V(s) \times \left( G_{21}(s)V(s) + G_{22}(s)C(s) \right) \]

(20)

From matrix MIMO system defined in (5) it is easy to see that the interactions are now governed by the product \( V(s) \cdot C(s) \) and the self-interaction \( V(s) \cdot V(s) \). Because the lack of information about the self-interaction one can neglect such term

\[ V(s)V(s) \approx 0. \]  

(21)

In this way it is possible to define the variable \( N(s) \) in a manner much more simplified as written below


\[ N(s) = G_{11}(s)V(s) + G_{12}(s)C(s) + G_{11}(s)G_{22}(s)V(s)C(s) \]

where the interaction adds an extra term consisting of the product of up to 4 quantities in the Laplace space.

C. Usage of the Faltung Theorem

Once the products are well defined in the Laplace space, it is quite advantageous to use the Faltung Theorem in the sense that the product of two functions in the Laplace space can also be written as a convoluted integral. Mathematically speaking one can write the full interaction term from the Laplace space to the time,

\[ G_{11}(s)V(s)G_{22}(s)C(s) = \int_{0}^{t} g_{11}(t-\tau)V(\tau)d\tau \cdot g_{22}(t-\tau')C(\tau')d\tau' \]

(23)

where the transfer functions appear to be delayed as noted in the integrals. In addition, expression (8) is carefully defined in order to extract a Volterra-like term which would give account of the memory of the system under certain circumstances.

D. Apparition of the Volterra-like Integral

From (23) the variables interaction term is defined as the product of two convolutions. It is also admissible to rewrite again (8) in the manner as given below [4],

\[ = \int_{0}^{t} \int_{0}^{t} g_{11}(t-\tau)g_{22}(t-\tau')V(\tau)C(\tau')d\tau d\tau' \]

(24)

where one can observe the nonlinear character of the interaction term. On the other hand, we performed the next changes

\[ -\tau' = \beta_1 \Rightarrow -\tau' = t - \beta \]
\[ -\tau = \beta_2 \Rightarrow \tau = t - \beta \]

(25)

Together to the product \( d\tau d\tau' = (d\beta_1)(-d\beta_2) = d\beta_1 d\beta_2 \). In this way one arrives to a double integral with their “input” functions which appears to be delayed,

\[ = \int_{0}^{t} \int_{0}^{t} g_{11}(\beta_1)g_{22}(\beta_2)V(t-\beta_1)C(t-\beta_2)d\beta_1 d\beta_2 \]

(26)

And which can be recognized as a second order Volterra-like term. Actually to speak about Volterra series in a consistent manner, \( C(s) = V(s) \). In one hand (26) changes the scenario to one most phenomenological: the inclusion of variable interactions brings us an extra term consisting in one which is featured by having memory (to some extent) [3]. It would have to be reflected on the permanent states of fluctuations and instabilities along the control horizon. Remarkably the interaction term would give a nonlinear piece to the mathematical formulation of the problem. In praxis Eq. (26) can be used for computational tests and simulations.

E. Full Expression in Time

The application of the Faltung theorem in (20) taking into account the Volterra-like term (24) would give a relation which should reflect the dynamics of a system under its temporal evolution, including its own interactions. Therefore the full expression corresponding to the output variable \( N(t) \) is written as

\[ N(t) = \int_{0}^{t} g_{11}(t-\tau)V(\tau)d\tau + \int_{0}^{t} g_{12}(t-\tau)C(\tau)d\tau + \int_{0}^{t} g_{11}(\beta_1)g_{22}(\beta_2)V(t-\beta_1)C(t-\beta_2)d\beta_1 d\beta_2 \]

(27)

Where the additional Volterra-like term appears as an extension of the linear case. It is possible to associate to this expression a coupling constant whose role would be that of tuning the interaction strength,

\[ N(t, \lambda) = \int_{0}^{t} g_{11}(t-\tau)V(\tau)d\tau + \int_{0}^{t} g_{12}(t-\tau)C(\tau)d\tau + \int_{0}^{t} \int_{0}^{t} g_{11}(\beta_1)g_{22}(\beta_2)V(t-\beta_1)C(t-\beta_2)d\beta_1 d\beta_2 \]

(28)

with \( \lambda = 0 \) the case without interactions is obtained. It is important to clarify that \( \lambda \) also appears as a tuning parameter of the action of the interaction to dynamics plant.

I. PLANT OF COMBINED CYCLES AND DATA

A. Plant Highlights

Data is extracted from a Power Plant Facility, located in Lima. The combined cycle processes is featured in providing an extra power of up to 75 MW. It is because the plant contains capabilities to convert heat in mechanical energy for feeding a secondary engine. A short explanation is as follows: plant uses two engines, one which is the main engine, aimed to generate 150 MW being fueled by nature gas, solely (gas comes from natural gas reservoir). This engine in the same time which is providing power electricity is also expelling heat around the 400 °C. This heat is used for heating water which is sent to a drum (it should be noted the importance in using this heat as a mechanism for avoiding local environment contamination). A valve regulates the water flux to the drum. Note the importance of the valve as an independent variable from a control system point-of-view. Consequently, the water vapor formed in the drum serves is to produces a vapor flux to push out the second engine. This second engine is capable to produce up to 75 MW. Thus, both engines are able to produce 225 MW. All this dynamics defines a combined cycle plant. In addition, the constant and stable production of oversaturated vapors is a must for the efficient functionality of the engines. Runs of the cycle combined plant might consider fluctuations in up to 0.5%. In table 1 some specifications of the most relevant variables used in this work are listed. In the present study, a concrete run is taped. Data is then stored for each variable during a run of 120 minutes. Beyond this time,
variables get stable with minor instabilities. Data taking considers a negligible systematic error. Because only a set of values are taken, no any statistical error is assumed.

<table>
<thead>
<tr>
<th>Plant Variable</th>
<th>Variable Specifications</th>
<th>Units</th>
<th>Type</th>
<th>Range</th>
<th>Setpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drum Level</td>
<td></td>
<td>Cm</td>
<td>Output</td>
<td>-20 to 30</td>
<td>0</td>
</tr>
<tr>
<td>Valve Position</td>
<td></td>
<td>%</td>
<td>Input</td>
<td>0 to 80</td>
<td>35</td>
</tr>
<tr>
<td>Water Flux</td>
<td></td>
<td>Kg/s</td>
<td>Input</td>
<td>0 to 100</td>
<td>56</td>
</tr>
<tr>
<td>Drum Pressure</td>
<td></td>
<td>Bar</td>
<td>Output</td>
<td>0 to 250</td>
<td>110</td>
</tr>
</tbody>
</table>

B. Variable Dynamics

The set points listed above actually correspond to the optimal values which had to be reached for an efficient functionality. For this run, no any alert was registered. Variables behavior are plotted in figure 1 where are displayed their individual evolution during the first 120 minutes as well. It should be remarked that the curves exposed in Fig. 1 share the same morphology during the first 35 minutes. However, the drum level curve is affected by the water flux and valve position. On the 45th minute the drum level tries to recover its set point reaching the level “0” around the 52th minute. Interestingly, the drum pressure becomes stable just in this time and appears to keep its stability over the upcoming minutes. The plot also indicates us that the time evolution of the valve position and water flux share approximately the same morphology as well. We claim that similar values of the valve position and water flux might give place to the presence of variable interaction by originating a dip on the drum level at the 44th minute. The evident irregularity that the drum level manifests over the upcoming minutes might have its origin in the instabilities of the valve position and water flux as well. The combined cycle plant targets to keep the drum level at its zero value during a complete run, without any strong fluctuations along the control horizon.

II. TESTING THE MODEL THROUGH NUMERICAL INTEGRATION

In order to test the semi-empirical model as given by the Eq. 13 we have assigned a physical meaning to the functions defined there. Thus \( V(\tau) \) the valve position, \( C(\tau) \) the water flux, and \( N(t, \lambda) \) the drum level whereas the parameter \( \lambda \) denoting the strength of interaction. In this way we write down the transfer functions

\[
 g_{11}(t) = \text{Heav}(t - 30.5) \ e^{-(t-30.5)} + e^{-(t-30.5)} \\
 g_{22}(t) = \text{Heav}(t - 20.5) \ e^{-(t-20.5)} + e^{-(t-20.5)}
\]

with \( \text{Heav}(t-r) \) the well-known Heaviside function, “r” a nominal number denoting the delay. In the Laplace space these transfer functions are found as

\[
 G_{11}(s) = \frac{3e^{-30.5s}}{1+3s} + \frac{5e^{-30.5s}}{2+5s}, G_{22}(s) = \frac{e^{-20s}(9+20s)}{(1+2s)(2+5s)}
\]

which would identify the system. In Fig. 2 is depicted the integral (28) for different values of \( \lambda \) [5]. For all plots is used Eq. (28) with an upper integration limit of up to 80 minutes. In a first sight one can see the effect of inserting the interaction by means the presence of two peaks and a flat behavior between them as observed on the upper panels (left / right). Note that in according to the these combined cycles processes \( N(t) = 0 \) [4].

So the dip found on data can be interpreted as the apparition of two sharp peaks as consequence of variable interaction. In this case the drum pressure and valve defines a type of interaction

![Figure 1](#).

![Figure 2](#).

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inside the drum [5]. It leads to modify roughly the morphology of the drum level to some extent. It is noteworthy that the presence of a dip might also be perceived as a transition phase between linear to nonlinear thereby changing the intrinsic system properties. While parameter $\lambda$ takes small values, the presence of a second peak turns out to be negligible as seen in lower left panel. For this case $\lambda = 0.0001$. It suggests that the drum level [6] goes to the case without interaction and a “tanh” shape is obtained. In figure 3 a full 3D plot is displayed for the drum level depending on $\lambda$ and $t$. Clearly the plot demonstrates that high values of $\lambda$ might be incoherent with data and plant phenomenology [7]. Therefore one can restrict to $\lambda$ values between 0 and 1 as dictated by the plant processes. In future a full 4×4 shall be studied to understand the phenomena which are not perceived directly in plant. All plots were obtained with the assistance of MAPLESOFT [4].

![Diagram of N(t,λ)](image)

Figure 3. (Top) The 3D modeling with Maple Soft[4] of the drum level as function of $\lambda$ and time according to Eq. 28. Note that the interaction increases when $\lambda$ is large. (Bottom) Evolution of the drum level variable from Eq. 27 that should be 0.

III. CONCLUSION

In this note, we have presented a pedagogical introduction about the usage of the theory of system identification applied to a concrete case of physical variables in a combined cycles power stations. For this end we have investigated the effect of variable interaction in a combined cycle power plant process for a single run. The numerical simulation yields some similarity to acquired data in the sense that the presence of a dip as noted in the drum level can be interpreted as the presence of two peaks on the nominal behavior during its temporal evolution. Indeed, an interaction parameter arises as a module which tunes the coupling of the interaction term to the linear case. This parameter is expected to be between 0 and 1 in order to be consistent to data. Therefore, variable interaction might be embedded in systems whose complexity overlaps the perception and might be a source of nonlinear behavior. This methodology might be of interest for those students whom are trying to relate a first course of systems with real applied engineering as is done in power plants.

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