

## Issues in the Blending of Curves for the Manufacture of Sculptured Surfaces

**A. Gittens**

*Research Assistant, Department of Mechanical and Manufacturing Engineering,  
The University of the West Indies, St. Augustine, Trinidad and Tobago, [agittens@eng.uwi.tt](mailto:agittens@eng.uwi.tt)*

**B.V. Chowdary**

*Lecturer, Department of Mechanical and Manufacturing Engineering,  
The University of the West Indies, St. Augustine, Trinidad and Tobago, [chowdary@eng.uwi.tt](mailto:chowdary@eng.uwi.tt)*

### Abstract

In sculptured surface modeling, complex contours are represented as a network of patches, each expressed in terms of known points, vectors, and curves. The contour of each patch conforms to that of the small surface section it is intended to represent. Their profiles may be approximated by determining their control points, and fitting a spline using an appropriate blending function. Together, the patches describe surface contours that would otherwise be difficult to define mathematically. In this study, complex shapes are defined as the sculptured surfaces created by the blending of curves, to form surface patches. Further, complex shapes formed by performing Boolean operations on geometrical primitives are beyond the scope of this paper.

In this research work, five general types of curves will be considered, namely: i) Parametric Polynomial, ii) Hermite, iii) Bezier, iv) B-Splines (Basis Splines), and v) NURBS (Non-Uniform Rational B-Splines). Curves created by these methods may be considered as the foundation of sculptured surface creation as the blending of any of these curves in three-dimensional space will form a sculptured surface patch. In this paper, some blending methods are investigated, and an example is selected to show how the blending functions contribute to the development of sculptured surfaces.

### Keywords

Sculptured Surfaces, NURBS, Blending Functions, Computer Aided Design and Manufacturing

## 1. Introduction

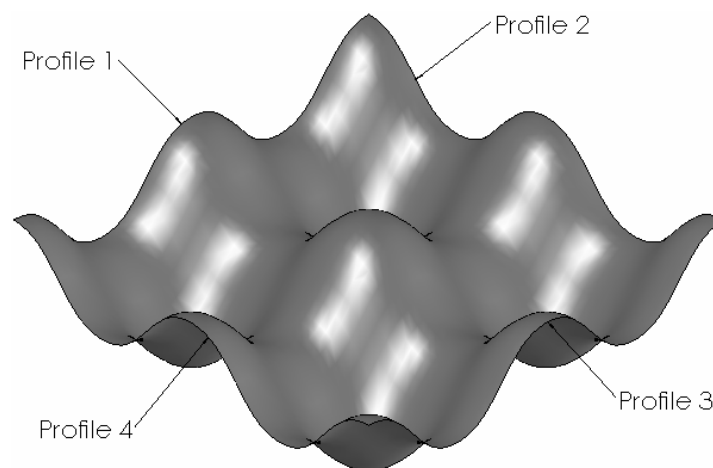
Most modern computer-aided design packages support the creation of free-form surfaces (Han and Yang, 1999). Algebraic and parametric rules generally govern the creation of these surfaces (Suresh and Yang, 1999). Sculptured surfaces are formerly considered to be impractical to produce with numerical control machining (Krouse, 1981). These surfaces are found in a wide range of components including those for aircraft, automobiles, construction and agricultural equipment, machine tools, home and office appliances, cameras, and instrument cases.

Current trends in consumer product design see the increasing use of freeform shapes. These sculptured surfaces are generated using commercial 3D modeling software packages such as Rhino®, CATIA®, Unigraphics®, ProE® and Solidworks®. CAM software (MasterCAM® and FeatureCAM®) is then used for generation of toolpaths for mold manufacturing using computer numerical control (CNC) machines. To accurately manufacture molds and achieve the tight tolerances required, several methods of toolpath generation are used (Baptista and Antune Simoes, 2000).

The creation of a sculptured surface is initiated by the selection of the appropriate control points for the required curve. Some common curve types are: i) Parametric Polynomial, ii) Hermite, iii) Bezier, iv) B-Splines (Basis Splines), and v) NURBS (Non-Uniform Rational B-Splines). Curves created by these methods would be considered as the foundation of sculptured surface creation as the blending of any of these curves in three-dimensional space will form a sculptured surface patch.

## 2. Objectives and Scope of Work

In this paper a Coons' surface type is considered for blending purposes. For illustration of the study the Coons' surface shown in Figure 1 is considered, which is made by blending of four drive profiles. The Coons' surfaces are quite powerful, but are often very difficult to create. They are named after Stephen A. Coons, who developed this surface type (Davis, 2002).



**Figure 1: Coons' Surface**

Five curve creation methods for design and development of the Coons' surface are selected in this study for investigation, with an objective of decreasing the machining time and improvement in surface finish of the blended surface under consideration. The methods to be examined are namely: i) Parametric Polynomial, ii) Hermite, iii) Bezier, iv) B-Splines and v) NURBS.

## 3. Methodology

To investigate the effect of surface type and machining influence on sculptured surfaces, two steps are suggested: i) selection of the construction curve to form the blended surface, and ii) simulation of the toolpath for machining of the blended surface.

### 3.1 Step 1: Selection of the construction curve

Five curve creation methods for design and development of the Coon's surface are reviewed. Each curve is briefed in the following sections with properties relevant to modeling and manufacture of sculptured surfaces.

#### 3.1.1 Parametric Polynomial Curves

According to Wang (2001), a parametric curve is a point-bound collection of points whose coordinates are defined by one parameter function  $u$ , of the form:

$$x = x(u), y = y(u), z = z(u)$$

Polynomial Parametric curves have  $C^0$  parametric continuity (Angel, 2002). This means that the endpoints match, but the tangent of the adjoining points may or may not match.  $C^0$  parametric continuity can be explained by considering the figure below. The curve  $p(0)$  to  $p(1)$  is joined to the curve  $q(0)$  to  $q(1)$ . This is shown in Figure 2. Due to this  $C^0$  parametric continuity, they are not the best choice for creation of sculptured surfaces, since the undesirable blending lines may still be visible after machining.

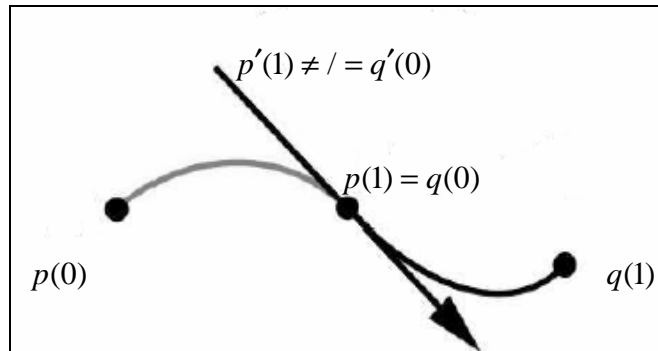


Figure 2:  $C^0$  Continuity

#### 3.1.2 Hermite Form

The Hermite Form as shown in Figure 4 uses two interpolating points and two derivatives (Angel, 2002). Considering the two derivatives allows for smoother joining of segments, as  $C^1$  parametric continuity is achieved. With reference to Figure 2, above,  $C^1$  parametric continuity would mean that:

$$p'(1) = \begin{bmatrix} p'x(1) \\ p'y(1) \\ p'z(1) \end{bmatrix} = \begin{bmatrix} q'x(1) \\ q'y(1) \\ q'z(1) \end{bmatrix} = q'(0) \dots\dots\dots (1)$$

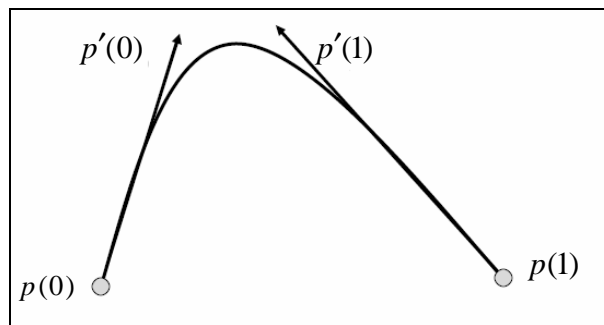
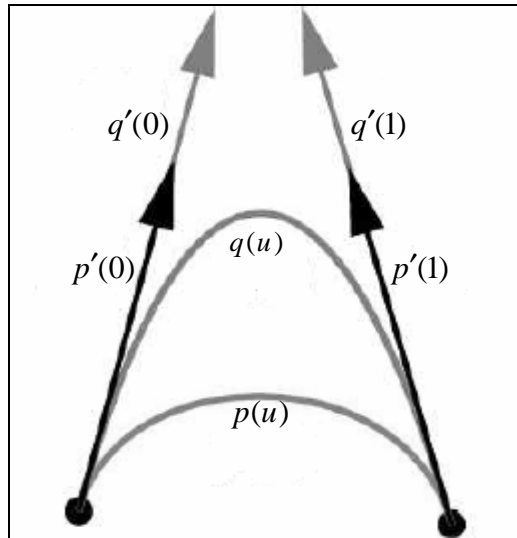


Figure 3: Hermite Curve

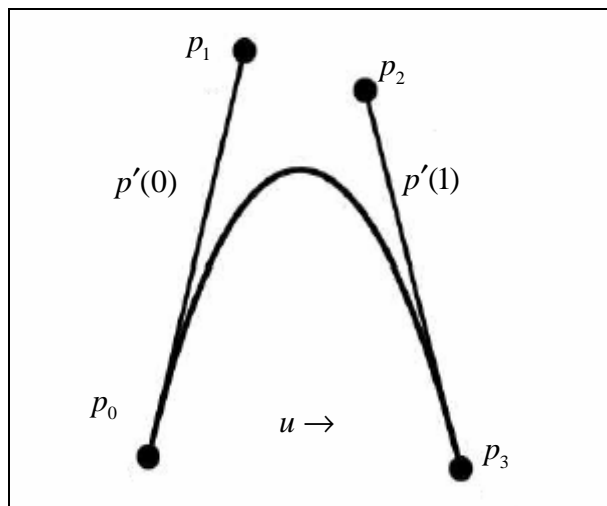
Hermite curves also have  $G^1$  continuity, which means that the tangents at the joints have same direction, but not necessarily the same magnitude. This can be seen in Figure 4, (Angel, 2002) where although the tangent directions for the curves at the endpoints are the same, the magnitude varies. This property is useful for drawing applications, as it gives more flexibility, thus leading to more controlled sculptured surfaces, and better machining possibilities.



**Figure 4:  $G^1$  Continuity**

### 3.1.3 Bezier Form

Bezier suggested that the derivatives of the Hermite curve could be approximated using the four data points of cubic interpolation (Angel, 2002).

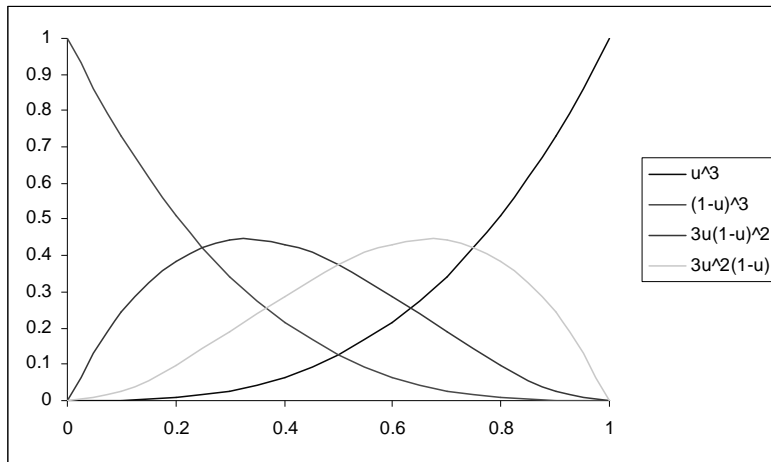


**Figure 5: Bezier Curve**

This is shown in Figure 5, where the locations of the points in the  $u$  direction are as follows:

$$p_0 @ u = 0, p_1 @ u = \frac{1}{3}, p_2 @ u = \frac{2}{3}, p_3 @ u = 1$$

This yields the Bezier blending curves for  $0 \leq u \leq 1$  as shown in Figure 6. Since Bezier curves are derivatives of the Hermite curve, they exhibit similar properties and hence also a good choice for sculptured surface creation.



**Figure 6: Bezier Blending Curves**

The four weights  $(1-u)^3$ ,  $3u(1-u)^2$ ,  $3u^2(1-u)$ , and  $u^3$  are known as Bernstein Polynomials. The general Bernstein form of Bezier curves is given by (Weisstein 2005, “Bezier Curve”) as:

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i} \dots\dots\dots (2)$$

Where  $\binom{n}{i} = \frac{n!}{i!(n-i)!}$  for  $n \geq i$  is a binomial coefficient.

### 3.1.4 B-Splines

B-Splines use data at a point,  $p = [p_{i-2} p_{i-1} p_i p_{i+1}]$  to define a curve between  $p_i$  and  $p_{i-1}$ . These curves allow us to have more continuity conditions than other forms, but usually require three times as much work in the case of cubic B-Splines.

Cubic B-splines may be represented by the equation,

$$p(u) = u M_s p \dots\dots\dots (3)$$

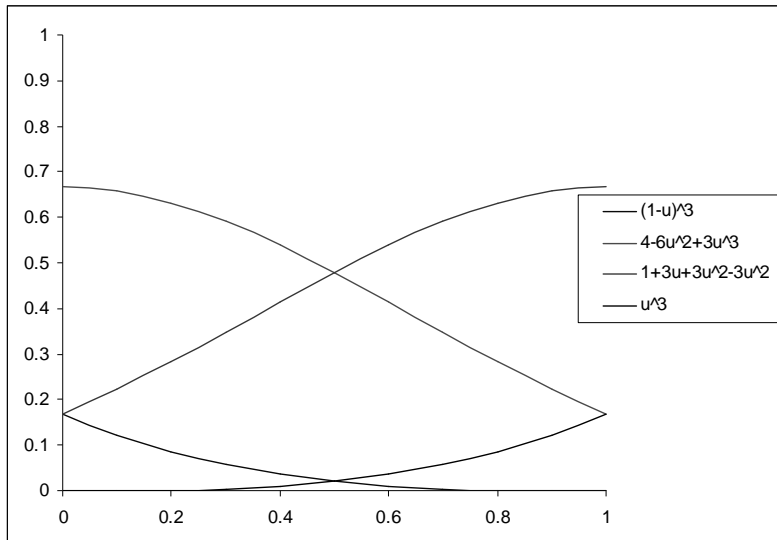
Where the B-Spline Matrix,  $M_s = \begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \dots\dots\dots (4)$

$$p(u) = B(u) p \dots\dots\dots (5)$$

Thus the blending function for this form is given as

$$B^{-1} = \frac{1}{6} \begin{bmatrix} (1-u)^3 \\ 4-6u^2+3u^3 \\ 1+3u+3u^2-3u^3 \\ u^3 \end{bmatrix} \dots\dots\dots (6)$$

The blending curves are shown in Figure 7.



**Figure 7: B-Spline Blending Curves**

Unlike other forms, first and second derivative continuity can be applied at the join points of B-Splines (Angel, 2002). This property gives B-Splines an advantage over Bezier, Hermite and Parametric curves for the modeling of sculptured surfaces.

### 3.1.5 NURBS

NURBS modeling is currently the most advanced mathematical model for splines. Non-Uniform Rational B-Splines add a fourth variable  $w$  to the standard  $x, y, z$  of curves. The  $w$ , is interpreted as the weight of importance given to some control value. Quadratics are a special case of NURBS. NURBS require perspective division, and act correctly for perspective viewing (Weisstein, 2005).

A NURBS Curve is defined by 
$$C(t) = \frac{\sum_{i=0}^n N_{i,p}(t)w_iP_i}{\sum_{i=0}^n N_{i,p}(t)w_i} \dots\dots\dots (7)$$

Where:  $p$  is the order.

$N_{i,p}$  are the B-spline basis functions

$P_i$  are the control points

$w_i$  is the weight of the last ordinate of the homogeneous point

NURBS are industry standard tools for the representation and design of geometry. Some reasons for the use of NURBS are, that they (Altmann, 2005):

- i.) Offer one common mathematical form for both, standard analytical shapes (e.g. conics) and free form shapes.
- ii.) Provide the flexibility to design a large variety of shapes.
- iii.) Can be evaluated reasonably fast by numerically stable and accurate algorithms.

iv.) Are invariant under affine as well as perspective transformations.

However, one of the drawbacks of NURBS is the need for extra storage to define traditional shapes (e.g. circles). This results from parameters in addition to the control points, hence allowing the desired flexibility for defining parametric shapes.

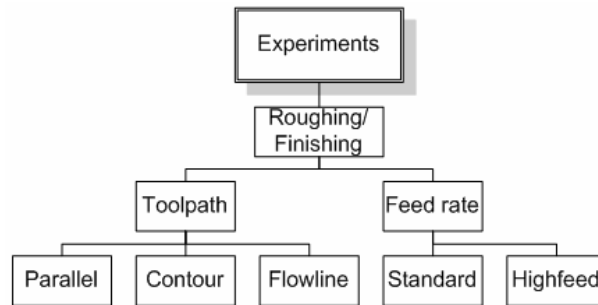
Since computer hardware has become more affordable in recent years, the NURBS file size, and the computer processing power required for their creation and modification is no longer an influencing factor for the selection of NURBS. Thus NURBS are selected for the generation of the Coons' surface in this study.

### 3.2 Step 2: Simulation of the toolpath

The machining of the sculptured surface shown in Figure 1 was simulated using MasterCAM 9.1®. The size of the stock used was  $120 \times 120 \times 50 \text{ mm}$ . with a  $9.525 \text{ mm}$  flat end mill for roughing, and a  $6.25 \text{ mm}$  ball end mill for the finishing operation.

The experiments done in this paper were designed based on Taguchi Methods. A  $(2^2 \times 3^2)$  orthogonal array of experiments was designed with the aid of Minitab 14® software.

In this study the factors chosen were the toolpath and feed rate for roughing and finishing operations, with the levels being parallel, contour and flowline for the toolpath factor, and standard and highfeed for the feed rate factor. This can be seen in Figure 8.

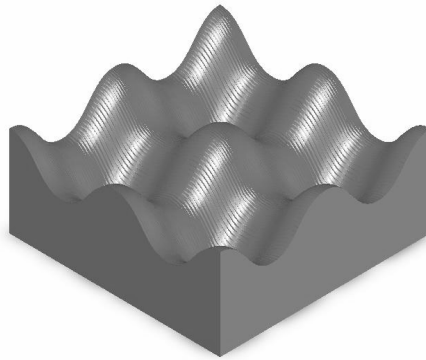


**Figure 8: Hierarchical Structure for Design of Experiments**

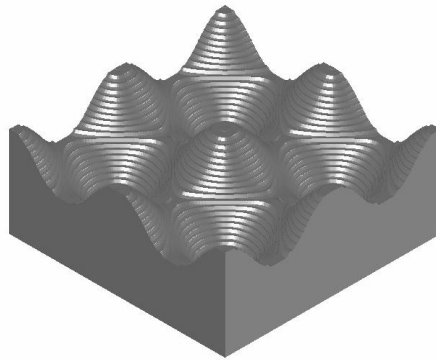
The surface finish and machining time are recorded for each experiment. This was done to allow for determination of the factors of the surface machining operation, which would lead to the best surface finish within the shortest period of time.

## 5. Results/Findings

The toolpaths were simulated using MasterCAM software. The machining time was recorded directly from MasterCAM, whereas the quality of the surface finish (weighted on a scale of 1-5) was given a value equal to the inverse of the length of the largest scallop, measured using RealVIZ Image Modeller 4.0® software. Figures 9 and 10 show simulated surface finishes attained through experimentation.

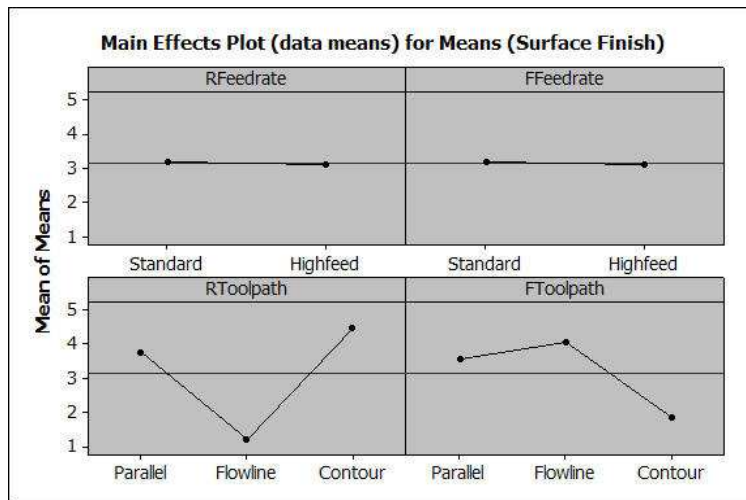


**Figure 9: Simulated Surface Finish for Contour Roughing and Flowline Finishing Toolpaths**



**Figure 10: Simulated Surface Finish for Contour Roughing and Contour Finishing Toolpaths**

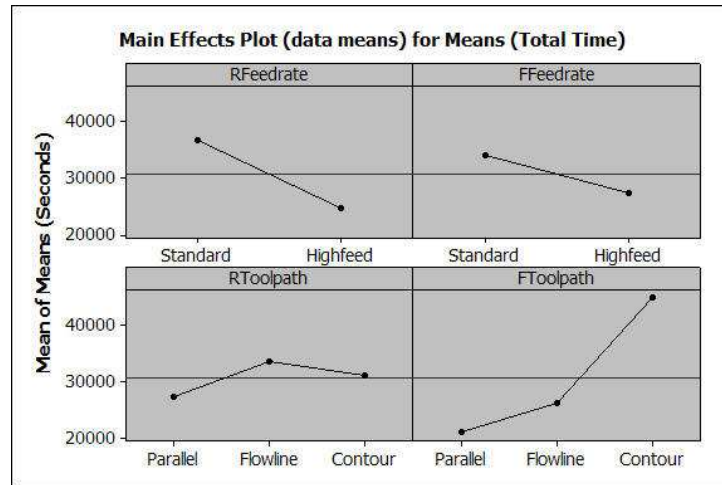
The summary of the experimental results of the study are shown in Figures 11 and 12.



**Figure 11: Main Effects Plot for Surface Finish**

From the Figure 11, it appears that the best surface finish for machining the Coons' surface was obtained by using the standard feed rate and a contour toolpath for roughing, while using standard feed rate and a flowline toolpath for finishing. The graph also shows that the worst finish was obtained by using the highfeed option with a flowline toolpath for roughing and a contour toolpath for finishing.





**Figure 12: Main Effects Plot for the Total Machining Time**

To decrease the roughing time, it is best to use the highfeed option and parallel toolpath. This is shown in Figure 12. The worst combination would be a standard feed rate with a flowline toolpath. To decrease the finishing time, it is again best to use a highfeed feed rate and a parallel toolpath. However the worst combination would be a standard feed rate with a contour toolpath.

## 6. Discussion

The results of the experiments show some interesting trends, in particular the differences between the output for roughing and finishing operations. It is apparent from the Figure 7, that the feed rate has a very small effect on the surface finish. This was expected, since the effects of tool vibration and deflection are present when there is unbalanced tool loading. Therefore it was expected that when the highfeed option is used, surface finish would be better. However, this is inconsistent with the trends on the graph. A possible reason for this may have been that the size of workpiece and tool used will not allow for the amplification and measurement of minor defects. Thus the results are seen in the graphs are conflicting, albeit very subtly.

The effect of varying the toolpath seems to be much greater than that of using different feed rates when considering the surface finish. This is expected, since the volume of material removed owing to tool vibration and deflection is much less than that of each tool step during machining.

The machining time was significantly affected by variations in both toolpath and feed rate for roughing and finishing. The results show that the feed rate selected has a much greater effect during roughing than in finishing. This can be deduced from Figure 12, where the gradient of the line for roughing is steeper than that for the finishing operation. This is probably owing to the difference in the volume of material being removed in each operation. Since a larger tool is used for roughing, it can be assumed that more material is removed per tooth, and more material is left uncut to prevent gouging. Thus there is likely to be more deviation in the value of the cutting tool's load.

The toolpaths selected had less effect on machining time during roughing than on finishing. This can be seen in Figure 12, where the spread of the points for roughing is much less than that of finishing. It must be noted though that time values given are based on simulation, and thus is more suited for comparison than for determining actual machining times. It is interesting to observe that the parallel toolpath gave the best machining times for roughing and finishing operations.

## 7. Conclusions

Based on the experimental results, it is possible to say that toolpath and feed rate selection are both significant factors in sculptured surface machining. For the workpiece selected, the toolpath selection seemed to have a greater effect on the finish and machining time than the feed rate.

It is important to note that the experiments were limited to the simulation of only one type of sculptured surface. Therefore, any generalisation about sculptured surface machining based on the results is not warranted. Another factor to consider is that the simulation process itself may be prone to errors, since the times output for some experiments seemed to be incongruous. However, for the purposes of this research, they were deemed suitable.

Auxiliary to the fact that the simulation results may have been flawed, it might be useful to undertake the actual machining of the surface. This should provide more accurate results, since measurements may be taken directly. It might be useful to record the force on the cutting tool during this time to determine if the algorithm used for obtaining highfeed machining accurately calculates the correct feed rate to produce a constant load. The work in this direction is in progress.

## References

- Altmann, M. (2006) About Nonuniform Rational B-Splines – NURBS, <http://www.cs.wpi.edu/~Ematt/courses/cs563/nurbs.html>, 04/10/06.
- Angel, E. (2002) Interactive Computer Graphics 3<sup>rd</sup>. Edition. Addison-Wesley, Ch. 10.1 – 10.6
- Baptista, R. and Antune Simoes, J.F. (2000) Three and five axes milling of sculptured surfaces, *Journal of Materials Processing Technology*, 103(3), pp. 398-403.
- Davis, C. (2002) *Mastercam® handbook volume 2, student guide*, San Diego, CA: San Diego CAD/CAM.
- Han, Z. and Yang, D.C.H. (1999) Iso-photograph based tool-path generation for machining free-form surfaces, *Journal of Manufacturing Science and Engineering, Transactions of the ASME*, 121(1), pp. 656-664.
- Krouse, J.K. (1981) Sculptured surfaces for CAD/CAM, *Machine Design*, 53(5), pp. 115-120.
- Suresh, K. and Yang, D.C.H. (1999) Constant scallop-height machining of free-form surfaces, *Journal of Engineering for Industry, Transactions of the ASME*, 116(1), pp. 253-259.
- Wang, X. (2001). Geometric Trimming and Curvature Continuous Surface Blending for Aircraft Fuselage and Wing Shapes. MSc. Thesis in Mechanical Engineering, Virginia Polytechnic Institute and State University.
- Weisstein, E. W. (2005) "Bézier Curve." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/BezierCurve.html>, 04/10/06.
- Weisstein, E. W. (2005) "NURBS Curve." From *MathWorld*--A Wolfram Web Resource. <http://mathworld.wolfram.com/NURBSCurve.html>, 04/10/06.

## Authorization and Disclaimer

Authors authorize LACCEI to publish the papers in the conference proceedings. Neither LACCEI nor the editors are responsible either for the content or for the implications of what is expressed in the paper.