# THE PROBLEM OF ENCODING-STRUCTURE IN FRACTAL ELECTRODYNAMIC ANTENNAS

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## INTRODUCTION

We present in this extended abstract a basic hypothesis of a Fractal Antenna with emphasis in their engineering application given a certain encoding scheme of the message signal. A brief effort has been done to give a unified and coherent treatment of the mathematical properties of geometric objects and their physical counterpart as a fractal antenna structure and the Electrodynamics Theory.

## FRACTAL OBJECTS

In approaching to a proper definition of Fractal Objects several aspects should be taken into account. First we may give a fractal description as a mathematical object with the following features:

- At arbitrarily small scales a fractal has affine structure
- It is too irregular to be easily described in traditional Euclidean geometric language. Most of the times non describable.
- It exhibits properties of self-similarity at least approximately or stochastically.
- It has a Hausdorff dimension which is greater than its topological (although this requirement is not met by space-filling curves such as the Hilbert curve).
- It has a simple and recursive definition.

A fractal is known to be a set of geometrical objects with an extended non-negative real number dimension that is in the closed interval $[0, \infty]$ associated with any metric space. This is called, the Hausdorff dimension of the geometrical object with the Hausdorff dimension strictly greater than its topological dimension.

By definition, the topological dimension of a set geometrical objects is always an integer and is 0 if it is totally disconnected, 1 if each point has arbitrarily small neighborhoods with boundary of dimension 0, and so on. Intuitively, the dimension of a set (for example, a subset of Euclidean space) is the number of independent parameters needed to describe a point in the set. One mathematical concept which closely models this naïve idea is that of topological dimension of a set. For example a point in the plane is described by two independent parameters (the cartesian coordinates of the point), so in this sense, the plane is two-dimensional.

As one would expect, topological dimension is always a natural number in the sense of the Lebesgue definition. In mathematics, the Lebesgue covering dimension of a topological space is defined to be the minimum value of $n$, such that any open cover has a refinement with no point included in more than n+1 elements. However, topological dimension behaves in quite unexpected ways on certain highly irregular sets such as fractals geometrical object which can present a high degree of fracturization. We may pose the Cantor set as an example of a geometrical object that has topological dimension zero, but in some sense it behaves as a higher dimensional space. The Hausdorff dimension gives another way to define dimension, which takes the metric into account.

To define the Hausdorff dimension for $X$, we first consider the number $N(r)$ of balls of radius at most $r$ required to cover $X$ completely. Clearly, as $r$ gets smaller $N(r)$ gets larger. Very roughly, if $N(r)$ grows in the same way as $1/r^d$ as $r$ is squeezed down towards zero, then we say $X$ has dimension $d$. In fact the rigorous definition of Hausdorff dimension is somewhat roundabout, since it first defines an entire family of covering measures for $X$. It turns out that Hausdorff dimension refines the concept of topological dimension and also relates it to other properties of the space such as area or volume.
It is appropriate to pronounce a word of warning is appropriate at this point. It is possible to define the'dimension' of a set in many ways, some satisfactory and others less so. It is important to realize that different definitions may

give different values of dimension for the same set, and may also have very different properties. Inconsistent usage has sometimes led to considerable confusion.

Hence to use the Hausdorff definition of dimension is at least proved to be sometimes unsatisfactory in that it excluded a number of sets that clearly ought to be regarded as fractals. Various other definitions have been proposed, but they all seem to have this same drawback.

Therefore when we refer to a set F as a fractal, we will have the following points as a enhanced description of a fractal object from the previous one:

- F has a fine structure, i.e. detail on arbitrarily small scales.
- F is too irregular to be described in traditional geometrical language, both locally and globally.
- Often F has some form of self-similarity, perhaps approximate or statistical.
- Usually, the ‘fractal dimension’ of F not necessarily of Hausdorff type (defined in some way) which is greater than its topological dimension.
- In most cases of interest F is defined in a very simple way, perhaps recursively.

## FRACTAL ANTENNA AS AN OBJECT

With the advance of wireless communication systems and increasing importance of other wireless applications, wideband and low profile antennas are in great demand for both commercial and military applications. Recent progress in the study of fractal antennas suggests some attractive solutions for using a single small antenna operating in several frequency bands. The term fractal, is used to describe a set or family of complex shapes which possess an inherent self-similarity or self-affinity in their geometrical structure. Fractal Electrodynamics has been open to study the interrelation between a propagating Electromagnetic Field and a huge fragmented object for the purpose of investigating and generating a new kind of radiation, the propagation of this new kind of radiation and the scattering and attenuation processes.

Recent efforts to combine Fractal Geometry with electromagnetic theory have led to new and innovative antenna designs. The engineering of this kind of mathematical objects is the design of the fractal antenna, and the application of the antenna at different scales. However as a metallic object has a resonant frequency with a electromagnetic wave the proper characterization of the antenna size is of primordial concern. These kind of antenna designs must possess some of the following attributes;

- Compact size
- Low profile
- Conformal
- Multiband or Broadband

The experimental and computational researches on such structures still lack a solid theoretical counterpart, which should yield a comprehensive framework for the definition and the analysis of vector electromagnetic fields defined on fractal objects subsets. As a consequence of this, some fundamental issues of electromagnetic theory for fractal radiators are still unsolved. Is it possible to radiate an electromagnetic pulse that matches the antenna structure eliminating the carrier signal and saving power at the transmitter?

## REFERENCES

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Fractal Antenna Awarded European Patent on Key Wireless Technology, December 6, 2004