# AN ANTENNA CALLED KOCH

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#### INTRODUCTION

Mathematics has always been concerned with sets of objects and functions to which the methods of classical calculus can be applied. However there are sets or functions that are not sufficiently smooth or regular and that largely have been ignored or not tackled thus, sometimes considered not worthy of study. Recently this attitude has changed and a great of interest has rised from this sets of mathematical objects called Fractals. Moreover, these irregular sets provide a much better representation of many natural phenomena than do the figures of classical geometry, and its material representation exhibits interesting features. Fractal geometry provides a general framework for the study of such irregular sets. In this letter, we'll use Fractal Geometry to design, pose the properties of a Koch Antenna and its theoretical problems.

## THE VON KOCH CURVE

If we have a line segment of unit length and we remove the middle third, the second set will consists of four segments, providing we replace the middle third by two segments of an equilateral triangle which base, is the removed segment. Figure 1 shows this process of consecutively removing the middle set from left side to the right.



Figure 1: Construction of von Koch curve

A simple calculation will yield that the k segment will have 
$$\left(\frac{4}{3}\right)^k$$
 of length.

### THE KOCH CURVE AS A FRACTAL ANTENNA

Benoît B. Mandelbrot (born 20 November 1924) is a French American mathematician, best known as the father of fractal geometry. He defined a fractal to be a set having Hausdorff dimension which is greater than the topological dimension.

Fractals are mathematical objects which can be used if the materialization of their geometry is done. They are remarkable in high frequency radio and also in mobile telephones. There are two advantages of using fractals. First, the 'space-filling' nature of some fractals, such as variations on the von Koch curve, allows a high response

fractal antenna to be fitted into a relatively small space. Second, as Benoît B. Mandelbrot defined a fractal to be a set having Hausdorff dimension which is greater than the topological dimension.

The meaning of this is that depending on their geometry, fractal antennas can be unique or multiband with resonant frequencies reflecting the parts that resemble the whole in some way, or alternatively can have frequency independent response. In the case of certain random fractal antennas, or with a higher level of complexity of self-similarity, (the whole in the part and vice-versa) to multiple frequency independent response.

## DESIGN, PROPERTIES AND THEORETICAL PROBLEMS OF A FRACTAL KOCH ANTENNA

There are various definitions of what a fractal is. If an object A, is similar to object B then object A or B, is a zoomed version of the other. A fractal is usually self-similar. That means that it repeats itself. For an example, look at the following fractal of Figure 2. In this fractal the original pattern repeats itself re-generating itself at the whole.



Figure 2: A von Koch fractal based on a very simple shape

Therefore, if an object is self-similar, then techniques for shrinking the size of an antenna through the use of fractals will preserve the properties and characteristics of a longer antenna. Fractal antennas can have radiation pattern and input impedances similar to those of longer antennae, yet take less area due to the many contours of the shape.

Using a fractal as a dipole antenna is essentially to miniaturize the total height of the antenna at resonance, meaning by resonance that the antenna has no imaginary component in the input impedance. The miniaturization of the fractal antenna is exhibited by scaling each iteration to be resonant at the same frequency.

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