Analysis of Impulsive Natural Phenomena through Finite Difference Methods – A MATLAB Computational Project-Based Learning

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Abstract - This paper is concerned with Project – Based Learning regarding the nature of the application of finite difference analysis in studying impulse response and vibration that occur in earthquakes. During many natural phenomena, oscillations occur that can be modeled by a second order differential equation. The finite difference technique has been implemented to determine the response of a spring-mass system, a mathematical model of one-degree-of-freedom frame structure, subjected to an impulsive force similar to one produced by an earthquake ground motion. Some attention has also been given when considering the application of various finite difference techniques; it is realized that the centered difference method is the best approach with the smallest error in analyzing the response of a vibrating system. Finally, a MATLAB script can be developed to generate impulsive response based on a finite difference numerical approach.

Keywords: Computational programming, Impulsive response, Finite difference methods, MATLAB, Earthquake analysis

I. INTRODUCTION

At its core, engineering has always been about the application of science and mathematics to innovate on century-old solutions, or to solve new age problems. In many cases, mathematical models have been constructed to describe the phenomena that is occurring, often resulting in complex second-order differential or partial differential equations. Obtaining meaningful solutions to these equations in closed – form solutions, often is not practical, whether it be known phenomena such as the heat equation, the wave equation or even describing a simple harmonic oscillator equation analytically. With one of the more important goals of engineers being to make things faster, more efficient and more accurate, it is important to have a medium through which students are exposed to a problem that allows them to make connections and demonstrate their knowledge while also improving their critical thinking skills with hands-on experience. With Project – Based learning this need is addressed, where complex real world problems can be tackled by students with oversight but still maintain the independent learning students will encounter in the workplace. For the purposes of this paper, we undertook a Project – Based Learning assignment using MATLAB to study vibration, and impulses occurring in earthquakes, while using finite difference analysis to simplify and study the mathematical description of this phenomenon.

Finite difference analysis provides an easy and effective alternative to analytical solutions. Having first modeled a situation with a governing equation and knowing all the variables except for the one which is being solved, the equation can be converted into a finite difference equation. Using simple equations derived from the Taylor's series involving forward, backward, and centered difference analysis, the governing equation can be converted into one that can be solved numerically using a MATLAB program. The simple nature of this conversion is probably one of the greatest advantages of this method as the resulting new equation does not need to be reformulated despite certain changes in specification, such as mass. The following is a direct application of finite difference technique to a second order differential equation governing impulse forces on a 1 – D mass which can be considered equivalent to those observed in earthquakes.

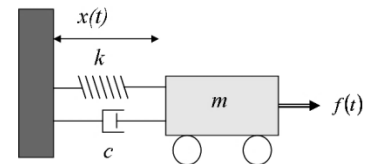## II. MOTIVATION OF GOVERNING EQUATION

Figure 1 Mass - Spring Force Diagram

Figure 1 represents a 1 – D mass – spring system which consists of,

$m - Mass\ (kg)$

$k - Spring\ contant\ /\ Material\ Stiffness\ \left(\frac{kg}{m}\right)$

$c - Damping\ constant\ (\frac{N-sec}{m})$ $f(t) - Impulsive\ Applied\ force\ (changes\ with\ respect\ to\ time)\ (N)$

$x(t) - Displacement(changed\ with\ respect\ to\ time)\ (m)$

From Newton's 2nd Law of Motion,

$$\sum F = ma; \qquad \text{Eq. 1}$$

Examining the forces acting on the mass, the following can be observed,

$$F_s : Force\ caused\ by\ spring$$

$$F_D : Force\ caused\ by\ damping$$

$$F(t) : Impulsive\ Applied\ force$$

Considering the x – direction to be positive pointing to the right, expansion of Equation 1 can be as follows,

$$-F_s - F_D + F(t) = ma$$

$$-kx - c\dot{x} + f(t) = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{f(t)}{m} \qquad \text{Eq. 2}$$

Equation 2 represents the governing equation that will be analyzed using finite difference methods to approximate the solution. Although, the problem is being analyzed in one dimension, there is still an inherent complexity regarding the forces acting on the mass. The main idea to be gleaned from this, is that earthquake ground motion produces an impulsive force on engineering structures similar to what we implemented in our analysis of mass-spring system.

Earthquakes result when sections of the earth, that normally move smoothly, stick to each other and cause energy to be stored. When the stored energy is finally great enough to break the two sections of earth free, the energy is released in seismic pulses to the surrounding areas.   This energy can be considered comparable to the applied force $f(t)$ in the mass – spring system. The seismic pulse effects of the earthquake are eventually damped out due to the energy dissipation which is analogous to the damping force of the mass – spring system. As a seismic wave propagates through the earth during an earthquake, the wave passes through the crust forcing it to rapidly accelerate and decelerate from the Primary waves, which cause vertical movement and Secondary waves that cause horizontal movement that you see in the Figure 2 [1]. The oscillations caused by these waves can be tracked by a similar harmonic oscillating system, such as the mass – spring system shown in Figure 3 [1], a fact backed up by the mass – springs system's use as seismographs to measure the intensity of earthquakes.
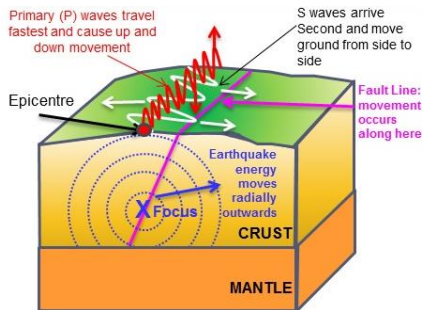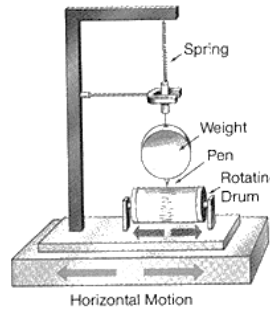

Figure 2 Earthquake Events Diagram

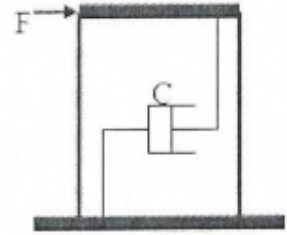Figure 3 Seismograph Mass – Spring system

Figure 4 One Story Frame Structure

In order to replicate earthquake ground motion we will analyze the impulsive response of a one story frame structure seen in Figure 4, which will be subjected to very sharp changes in force over short periods of time.

From the graph in Figure 4, a piecewise function can be constructed to represent the impulsive force present,

$$f(t) = \begin{cases} 17800(0.5 - t) & for\ \ t \le 0.5 \\ -33375(t - 0.5) & for\ \ 0.5 < t \le 0.7 \\ -22250(1 - t) & for\ \ 0.7 < t \le 1.0 \\ 26700(t - 1) & for\ \ 1.0 < t \le 1.5 \\ 13350(2.5 - t) & for\ \ 1.5 < t \le 2.5 \\ 0 & for\ \ 2.5 < t \le 10 \\ 10000 * (10 - t) & for\ \ 10 < t \le 10.5 \\ 10000 * (10 - t) & for\ \ 10.5 < t \le 11 \\ 0 & for\ \ t > 11 \end{cases}$$
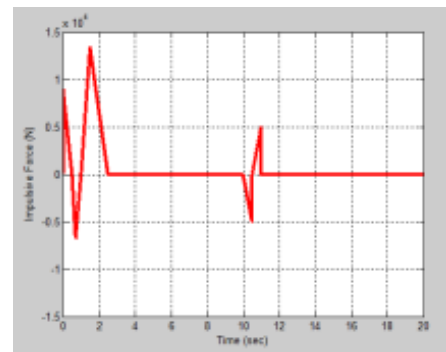

Figure 3 Impulse Graph

III. DERIVATION OF FINITE DIFFERENCE APPROXIMATIONS

First Order Forward in Velocity and Backward in Response (FVBR)

From Equation 2, the governing equation can be seen as,

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{f(t)}{m}$$

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Decomposing Equation 2 into a series of first order derivatives, it can be simplified to,

$$\frac{d}{dt}\left(\frac{dx_i}{dt}\right) + \frac{c}{m}\frac{dx_i}{dt} + \frac{k}{m}x_i = \frac{f(t)}{m} \qquad \text{Eq. 3}$$

The Forward Difference approximation of the 1st Derivative (derived in Appendix A) is,

$$f'(x_i) = \frac{f(x_{i+1})-f(x_i)}{h} + O(h) \qquad \text{Eq. 4}$$

Applying Equation 4 to Equation 3,

$$\frac{d}{dt}\left(\frac{x_{t_{(i+1)}}-x_{t_{(i)}}}{h}\right) = \frac{f(t_i)}{m} - \frac{c}{m}\frac{dx_{t_{(i)}}}{dt} - \frac{k}{m}x_{t_{(i)}} \qquad \text{Eq. 5}$$

Simplification of Equation 5 leads to,

$$\frac{dx}{dt}_{t_{(i+1)}} = h*\left[\frac{f(t_i)}{m} - \frac{c}{m}\frac{dx_{t_{(i)}}}{dt} - \frac{k}{m}x_{t_{(i)}}\right] + \frac{dx_{t_{(i)}}}{dt} \qquad \text{Eq. 6}$$

The Backward Difference approximation of the 1st Derivative (derived in Appendix B) is,

$$x'(t_i) = \frac{x(t_i)-x(t_{i-1})}{h} + O(h) \qquad \text{Eq. 7}$$

Applying Equation 7 to the $\frac{dx}{dt}$ in Equation 6, the finite difference approximation is finalized at,

$$x_{t_{(i+1)}} = h^2 *\left[\frac{f(t_i)}{m} - \frac{c}{m}v_{t_{(i)}} - \frac{k}{m}x_{t_{(i)}}\right] + h*v_{t_i} + x_{t_{(i)}} \qquad \text{Eq. 8}$$

Equation 5 also becomes the function to calculate v_{t_i},

$$v_{t_{(i+1)}} = h*\left[\frac{f(t_i)}{m} - \frac{c}{m}v_{t_{(i)}} - \frac{k}{m}x_{t_{(i)}}\right] + v_{t_{(i)}} \qquad \text{Eq. 9}$$

Because Equation 9 is derived from the Forward Difference approximation for the 1st derivative, it inherently needs an initial condition [2] (v_0). However, since Equation 8 contains two finite difference approximations, it needs two initial conditions. It turns out that one of the initial conditions overlaps because Equation 9 is imbedded within Equation 8.

Centered Difference Method

The centered difference method is very similar to the first method in that the derivation process is exactly the same. However, the two methods vary in their final formulas and their order of error.

Starting from Equation 2, the governing equation is the same,

$$\frac{d^2x}{dt^2} + \frac{c}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{f(t)}{m}$$

However, substitute Equation 4 with the centered difference formula for 1st and 2nd derivatives (derived in Appendix C),

$$f'(x_i) = \frac{f(x_{i+1})-f(x_{i-1})}{2h} + O(h) \qquad \text{Eq. 10}$$

$$f''(x_i) = \frac{f(x_{i+1})-2f(x_i)+f(x_{i-1})}{h^2} + O(h^2) \qquad \text{Eq. 11}$$

Substituting Equations 10 and 11 into Equation 2, the following relationship is formulated,

$$x_{t_{(i+2)}} = \frac{1}{1+\frac{m}{hc}}\left[\frac{m}{hc}\left(2x_{t_{(i+1)}} - x_{t_{(i)}}\right) - \frac{kh}{c}\left(x_{t_{(i+1)}}\right) + \frac{f(t)h}{c} + x_{t_{(i+1)}}\right] \qquad \text{Eq. 12}$$

As one can see, the final form of the centered difference method is completely unlike that of the forward difference method, and backwards difference method. The principal difference between the two methods is that the Forward-Backwards (FVBR) method allows for response and velocity to be calculated, whereas the centered difference approximation only evaluates for response. Due to this fact, it is also known that the initial conditions vary. For the FVBR method, an initial position and an initial velocity are needed. In contrast, as seen in Equation 12, the centered difference method needs the first two positions, and no velocity condition.

APPLICATION OF FINITE DIFFERENCE METHODS

Based on Equations 8 and 9, a MATLAB script can be written as follows to calculate the response as well as the velocity of the system being studied using the Forward in Velocity Backwards in Response Method,

Simply ask the user for the specifications of the system. Each variable is clearly described by its respective input message

Define the start of the three variables that need to be tracked, x, v and t, response, velocity and time respectively

Compute the correct forcing term which is based on time. Each of the five intervals of the piecewise function is expressed as a condition for a conditional statement to compute a different forcing term based on the given impulsive force

Calculate the next term in each of the response, velocity and time sequences respectively. As one can see, the finite difference method is applied here

Display the data to the user and plot time versus response.

Figure 4 Forward in Velocity / Backward in Response Script

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```
%% Impulsive Vibration Analysis, Earthquakes
% Central Difference Method

m  = input('Mass =            ');
k  = input('Spring Constant =  ');
c  = input('Damping Constant = ');
h  = input('Step Size =        ');
ti = input('Initial Time =     ');
tf = input('Final Time =       ');
xi = input('Initial Displacement = ');
vi = input('Initial Velocity = ');

i = 1;
x(1) = xi;          % Response vector
x(2) = xi;
t(1) = ti;          % Time vector
t(2) = ti + h;

while(1)
    t(i+2) = t(i+1) + h;
    if (t(i+1) <= .5)
        F(i+1) = 17800*(0.5 - t(i+1));     % Force term for the first 0.5s
    elseif (t(i+1) > 0.5 && t(i+1) <= 0.7)
        F(i+1) = -33375*(t(i+1) - 0.5);    % Force term for 0.5 < t <= 0.7
    elseif (t(i+1) > 0.7 && t(i+1) <= 1.0)
        F(i+1) = -22250*(1 - t(i+1));      % Force term for 0.7 < t <= 1.0
    elseif (t(i+1) > 1.0 && t(i+1) <= 1.5)
        F(i+1) = 26700*(t(i+1) - 1);       % Force term for 1.0 < t <= 1.5
    elseif (t(i+1) > 1.5 && t(i+1) <= 2.5)
        F(i+1) = 13350*(2.5 - t(i+1));     % Force term for 1.5 < t <= 2.5
    elseif (t(i+1) > 2.5)
        F(i+1) = 0;                        % Force term for t >= 2.5
    end

    x(i+2) = (1/(1+(m/(h*c)))) * ((m/(c*h)) * (2*x(i+1)-x(i)))...
        - ((k*h)/c)*x(i+1)) + (F*h)/c + x(i+1));
    if t(i+2) > tf, break, end;
    i = i+1;
end

disp([t' x'])
plot(t, x, 'b')
xlabel('Time (sec)')
ylabel('Response (m)')
```

Simply ask the user for the specifications of the system. Same as Previous Script

Define the start of the two variables that need to be tracked, x and t, response and time respectively (Note: there are two initial positions)

Compute the correct forcing term which is based on time. Same as Previous Script

Calculate the next term in each of the response and time sequences respectively. As one can see, the finite difference method is applied here

Display the data to the user and plot time versus response.

Figure 5 Centered Difference Script

The FVBR script calculates both the response and velocity of the column due to the earthquake. The script is driven by the incrementing of 'i' to calculate new terms while saving the previous points in a matrix. At the end, each matrix holds data from t = initial value to t = final value. The finite difference method derived in the previous section is defined by lines 34 and 35 of the script.

Based Equation 12, a MATLAB script can be written as follows to calculate the response of the system being studied using the Centered Difference Method which can be seen in Figure 6.

The Centered Difference script calculates only the response of the column of the one story structure during the earthquake. The script is driven by the incrementing of 'i' to calculate new terms while saving the previous points in a matrix. At the end, each matrix holds data from t = initial value to t = final value. The finite difference method derived in the previous section is defined by lines 37 and 38 of the script.

MATLAB FINITE DIFFERENCE RESULTS

Referring back to the problem shown in the 'Motivation of the Governing Equation' section, let us apply the equations to this situation.

Table 1 Initial Conditions

Case 2 – Average Step Size		Case 3 – Smallest Step Size	
Mass =	100	Mass =	100
Spring Constant =	12000	Spring Constant =	12000
Damping Constant =	20	Damping Constant =	20
Step Size =	.001	Step Size =	.0001
Initial Time =	0	Initial Time =	0
Final Time =	20	Final Time =	20
Initial Displacement =	0	Initial Displacement =	0
Initial Velocity =	0	Initial Velocity =	0

The conditions in Table 1 describe a column with a stiffness constant of 12,000 kg/m that is initially displaced 0 m and at rest. A sudden earthquake occurs with the impulse force shown in Figure 4. The column is studied from the time of impact to 20 seconds later.

Mixed (Forward in Velocity – Backwards in Response) Finite Difference

Table 2 Response values for Specific Time Mixed Difference

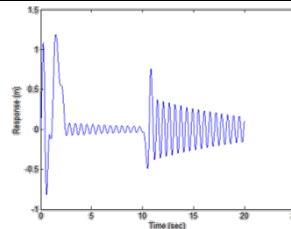| Time (s)	Response from 0.001 time interval (m)	Response from 0.0001 time interval (m)	Velocity from 0.001 time interval (m)	Velocity from 0.0001 time interval (m)
0	0	0	0	0
2.5000	−5.4491	−5.5041	57.3748	56.9099
5.0000	5.8299	5.8142	7.7392	8.2481
7.5000	−2.4257	−2.3672	−42.4052	−42.6115
10.0000	−1.1515	−1.2034	36.9887	36.8132
12.5000	2.6030	2.6167	−10.6397	−10.2807
15.0000	−1.8603	−1.8380	−11.9753	−12.2394
17.5000	0.2499	0.2165	18.2242	18.2465
20.0000	0.8827	0.9023	−10.6506	−10.4822

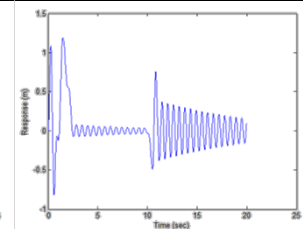*Figure 6 FVBR Method (0.001 time step)*

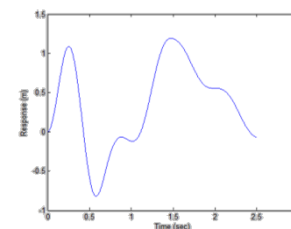*Figure 9 FVBR Method (0.0001 time step)*
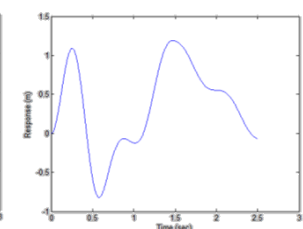

Figure 10 FVBR Method 2 (0.001 time step)

Figure 11 FVBR Method 2 (0.0001 time step)

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Centered Finite Difference

Table 3 Response values for Specific Time Centered Difference

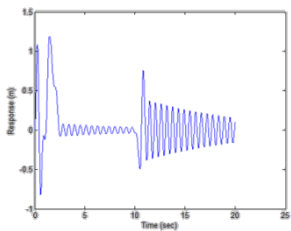| Time (s)	Response from 0.01 time interval (m)	Response from 0.001 time interval (m)	Response from 0.0001 time interval (m)
0	0	0	0
2.5000	−4.6731	−5.4624	−5.5054
5.0000	5.8280	5.8329	5.8144
7.5000	−2.8840	−2.4012	−2.3648
10.0000	−0.7062	−1.1855	−1.2068
12.5000	2.4432	2.6180	2.6182
15.0000	−1.9693	−1.8457	−1.8365
17.5000	0.4501	0.2203	0.2136
20.0000	0.7533	0.9037	0.9044

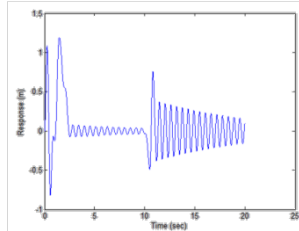*Figure 12 Centered Difference 0.001 time step*

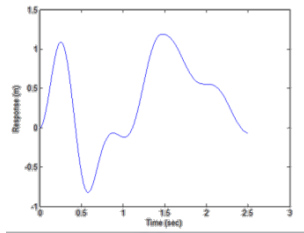*Figure 13 Centered Difference 0.0001 time step*

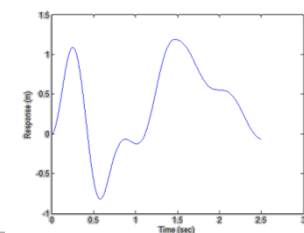*Figure 14 Centered Difference 2 (0.001 time step)*

Figure 15 Centered Difference 2 (0.0001 time step)

This data reveals that the column would need to be able to sustain a response range of about 18 m (given no energy absorption into the column) in order to withstand an earthquake this size. The graphs in Figures 7, 8, 11 and 12 display the response needed by the column in the first 20 seconds of the earthquake using two difference finite difference methods with two step sizes. Figures 9, 10, 13 and 14 examine the response needed in the first 2.5 seconds of the earthquake. All of the graphs are approximately the same which shows the overall accuracy of the finite difference methods; however, from the data in Tables 2 and 3, it is evident that there is some error. Figure 15 shows a typical acceleration vs. time graph for an earthquake[4]. Acceleration is simply the second derivative of response. Since the graph of acceleration can be represented as sine's and cosine's, the form of acceleration and response should follow suit. The graph in Figure 15 has a longer earthquake impulse force; nevertheless, the response is similar. Using the 1940 El Centro Earthquake Graph [4] as an experimental reference, the finite difference seismic graphs and the actual seismic graphs can be related

proving the data from the finite difference methods as accurate representations of the proposed situation.
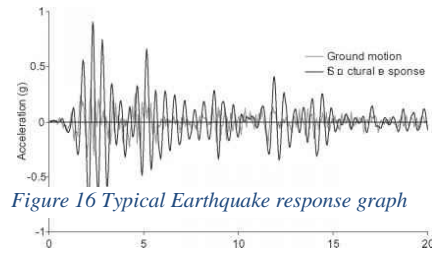

Figure 16 Typical Earthquake response graph

ERROR ANALYSIS

From the derivation of each finite difference method, it was shown that each approximation contains their share of error. Both the Forward and Backwards Difference Methods had an error in the order of h while the Centered Difference has an error in the order of h^2. The response studied in this application contains very sensitive data which changes in the hundredths place. As a result, a small step size had to be chosen in order to accurately represent the response of the column to the earthquake. Looking at the difference between the response from a 0.01 time step, a 0.001 time step and a 0.0001 time step, it can be seen that the data eventually converged to the actual value as a smaller time step was considered.

There is no analytical data to compare to the finite difference data. Nevertheless, knowing that the data should represent the actual analytical data, an approximate error can be calculated to examine the error between the two finite difference methods.

Approximate error can be calculated as follows,

$$\% \, Error = \left| \frac{Experimental \, Value_1 - Experimental \, Value_2}{Experimental \, Value_1} \right| * 100\%$$

Eq. 13

Applying this equation to the 0.0001 time step data for response for both methods provides the following results,

Table 4 Error Analysis between Step Sizes

Time (s)	1 - Centered Method Response from 0.001 time interval (m)	2 - Centered Method Response from 0.0001 time interval (m)	Approximate Error (%) between 1 and 2
0	0	0	00.000
2.5000	−5.4624	−5.5054	-0.787
5.0000	5.8329	5.8144	0.317
7.5000	−2.4012	−2.3648	1.516
10.0000	−1.1855	−1.2068	-1.797
12.5000	2.6180	2.6182	-0.008
15.0000	−1.8457	−1.8365	-1.046
17.5000	0.2203	0.2136	3.041
20.0000	0.9037	0.9044	-0.077

Table 5 Error Analysis between Finite Difference Methods

Time (s)	FVBR Method Response from 0.001 time interval (m)	Centered Method Response from 0.0001 time interval (m)	Approximate Error (%)
0	0	0	0.000
2.5000	−5.5041	−5.5054	-0.023
5.0000	5.8142	5.8144	-0.003
7.5000	−2.3672	−2.3648	0.101
10.0000	−1.2034	−1.2068	-0.283
12.5000	2.6167	2.6182	-0.057
15.0000	−1.8380	−1.8365	-1.469
17.5000	0.2165	0.2136	1.339
20.0000	0.9023	0.9044	-0.233

Note: The 0.0001 time step data was used in error calculated because it most accurately represents the true data.

Both methods present very accurate data with an error percentage of under 2% for the smallest time step. Table 4 shows that the 0.01 time step provided a very large error of 104.3% which is beyond unacceptable, especially for an application such as earthquake. Because the impulsive force is so short and the oscillations are so close, a very small time step needs to be chosen to account for the high frequency. There is a tradeoff between accuracy and efficiency when considering step sizes. For example, the 0.01 step size created 2000 data points for 20 seconds, whereas the 0.0001 step size created 200,000 data points. For a more complex application, considering all three dimensions of motions, such calculations would become memory consuming and inefficient for a computer; thus, the need to choose a larger time increment. Table 5 demonstrates a small error proving the accuracy of both methods.

REAL WORLD APPLICATION: GROUND MOTION AND BUILDING FREQUENCIES

When studying the effects the produced impulsive forces have on engineering structures, we see certain characteristics emerge that have a large influence on building response. These characteristics include duration, amplitude (of displacement, velocity and acceleration) and frequency of ground motion.

As it can be imagined, the motions of the building are very complex, almost as complicated as the ground motion itself. The primary issue engineers worry about however, is the building's tendency to vibrate around one particular frequency,
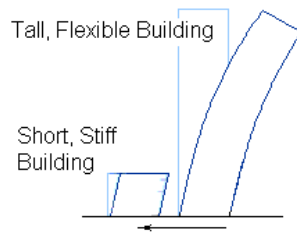

Figure 17 Building Comparison

known as its natural or fundamental frequency. Generally speaking, the frequency buildings vibrate at have a lot to do with their height. The shorter a building is, the higher its natural frequency. The taller the building is, the lower its natural frequency which can be seen in Figure 16.

This means that a short building with a high natural frequency also has short natural period. A very tall building with a low frequency has a long period. However, in reality, these numbers are quite short where a typical one story building has a period of 0.1sec. Although the natural period of buildings tend not to change, other factors such as loose ground soil, and cracking structures can have the effect of amplifying waves. Now you may be inclined to think that these waves alone result in large movements that cause huge amounts of damage. This is only partially true. In reality, the movement of the ground and buildings during an earthquake is not much regardless of earthquake size. So it is not the distance that the building moves that causes damage, it is the sudden force that causes the building to shift quickly that causes the damage. In other words, the damaging force is *acceleration*. During an earthquake, the speed at which both the ground and building are moving will reach some maximum. The more quickly they reach this maximum, the greater their acceleration, the greater the damage.
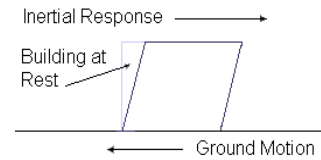

Figure 18 Dynamic Analysis

Dynamically this is related to Newton's second law and D'Alembert's Principle, which states that a mass acted upon by an acceleration tends to oppose that acceleration in an opposite direction and proportionally to the magnitude of the acceleration as shown in Figure 17. This in layman's terms means the larger the mass and/or the greater the acceleration, the larger the resulting force. When this force F is imposed upon the building's structural elements, beams, columns, load-bearing walls, floors, as well as the connecting elements that tie these various structural elements together, if they are large enough, cause the building's structural elements to suffer damage of various kinds.
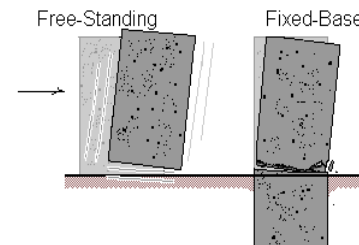

Figure 19 Support Types

In Figure 18 we see an example of the damage that can result from the acceleration caused by the earthquake ground motion. Assuming the free standing block is simply sitting on the ground without any attachment to it, the block will move freely in a direction opposite to the ground motion, with a force

proportional to the mass and acceleration of the block [5]. If the same block, however, is solidly grounded and no longer able to move freely, it must in some way absorb the inertial force internally. In Figure 18, this internal uptake of force is shown to result in cracking near the base of the block. As mentioned earlier this has a tendency to amplify waves which increases the severity of the damage, especially when the natural frequency of the building is equivalent to that of the earthquake, which causes resonance to occur along with a continuous absorption of energy until the structure collapses. The reason resonance is so destructive is the transfer of large amounts of energy causes large increases in the amplitude of a given wave, resulting in a structure that is unable to dampen the oscillations effectively and thus vibrating until the energy in the system forces a collapse.

The potential damaging effects of resonance can be seen in the oscillations of the famous Tacoma Bridge collapse. Opened on July 4, 1940 the bridge during its four months of active life was observed to have many transverse modes of vibration. A maximum crest to trough amplitude was observed to be about 5ft; the frequency of vibration at that time was 12 vib/min. On the morning of November 7 the wind velocity was 40 to 45 mph, Traffic was shut down and amplitude of about 3ft was observed. At about 10:00 a.m., the main front began to vibrate torsionally in 2 segments with a frequency 14 vib/min. The amplitude of the torsional vibration quickly built up to about 35 vib/min in each direction from the horizontal peaking, until just after 11 am when the left side of the bridge was 28ft higher than the right. At this point the bridge twisted and broke. In this situation, the applied force, caused by the wind, continually added energy to the system allowing for zero damping until the structure failed. The net outcome from the bridge demonstrates the destruction oscillations can cause [6].

When studying structural natural frequencies, we see that it is important that we have a good idea of the regional natural frequencies of earthquakes. As evident from the Tacoma bridge collapse, knowing the potential oscillation of any natural phenomena, and avoiding potential resonance can not only prevent disasters but save lives. Mathematically, these frequencies can be modeled as eigenvectors and eigenvalues from second order differential equations using finite difference technique. Although the application of this is beyond the scope of this paper this is just one more area where finite difference technique can be used to glean useful information applicable to earthquake motions effects on buildings.

Figure 20 Tacoma Bridge Collapse [7]

CONCLUSION

Knowing more about the dangers of earthquakes and the potential hazards that they create for us we now can see why it is important for us to have useful methods for analysis of engineering structures. With the use of finite difference methods for analysis we are able to replicate the impulsive forces produced by earthquake ground motion and as a result generate useful data for how they act on engineering structures using a simple mass-spring system. Although potentially far more complex than our analysis seen here for 1D motion, this technique has useful applications in 3D motion as well as in eigenvalues and eigenvectors modeling the frequencies during which destructive resonance can occur. Although those applications are beyond the scope of this paper, it can be seen that finite difference has many useful real world applications allowing otherwise numerically challenging models to be accurately approximated using simple forward, backward and centered differences. The MATLAB code solidifies this theory by allowing for fast, reliable and continual computations.

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