Consistent Conjectures in a Bilevel Human Migration Model

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Abstract*– We develop a bilevel human migration model using the concept conjectural variations equilibrium (CVE). The upper-level agents can be interpreted as the governments of emulating locations, whose strategies are amounts of investment into infrastructures of the municipalities (cities, towns, etc.). These investments aim at making the towns more attractive to both the residents and potential migrants from other places, as well as prospective businesses. At the model's lower level the current residents (considered as professional communities) are also potential migrants to other cities. They make their decision where to migrate (if at all) comparing the expected values of the utility functions of the outbound and inbound municipalities taking into account their group's conjectures concerning equilibrium migration flows between the involved towns. The utility functions reflect the conjectural variations technique since their values are based on the potential migrants flows affecting to the target locations' attractiveness. Applying a special criterion to verify the consistency of the conjectures (influence coefficients), the existence and uniqueness results for the consistent conjectural variations equilibrium (CCVE) are established.

***Keywords*—Conjectural variations equilibrium, influence coefficients, bilevel human migration model.**

I. INTRODUCTION[1]

Migration problems have boasted active studying throughout the world since the migration prediction data is extremely important on a large economic scale. Migration movement data can stipulate development of facilities necessary to advance employment, education, ecology, etc. Reciprocally, cities with more advanced infrastructure, higher employment capacities, ecology-friendly environment, etc., can attract large arrays of potential migrants. On the other hand, overloaded housing/infrastructure facilities are able to reduce the comfort of everyday life thus prevent people from moving to the town affected. Therefore, one can expect a tradeoff in the investments into a city's infrastructure and the expected migration flows directed to the municipality in question.

The list of existing and developing human migration theories has inflated over time. In the short historical aspect, one may rely on the excellent fundamental survey [1], as well as an interesting paper [12].

In papers [2]–[3], a new concept of conjectural variations equilibrium (CVE) was introduced and investigated, with the *influence coefficients* of each agent affecting the structure of the Nash equilibrium. In particular, constant conjectural influence coefficients were involved in the human migration model examined in [4]. More precisely, the potential migration groups were taking into account not only the current difference between the utility function values at the destination and original locations but also the possible variations in the utility values implied by the change of population volume due to the migration flow. These conjectured variations could be described with an aid of the so-called *influence coefficient* w_i . In other words, we considered not the *perfect competition* (with $w_i = 0$) but a generalized Cournot-type model with influence coefficients w_i in general distinct from 1 (in contrast to the classical Cournot model, where $w_i = 1$).

In their previous papers [5]–[6], the authors extended the latter model to the case where the conjectural variations coefficients may be not only constants but also functions of the total population at the destination and of the group's fraction in it. Moreover, we allow these functions to take distinct values at the outbound town and at the inbound place (destination), which elevated the model's flexibility. As an experimental verification of the proposed model, the authors developed a specific form of the model based on relevant population data of a three-city agglomeration at the boundary of two Mexican states: Durango and Coahuila. In more detail, the 1980–2005 dynamics of population growth in the three cities: Torreón (Coahuila), Gómez Palacio (Durango) and Lerdo (Durango) was considered, and utility functions of three various kinds for each of the three cities were exploited. To the best of our knowledge, utility functions of these types weren't used in the previous literature dealing with the human migration models. After having collected necessary information about the average movement and transportation (i.e., migration) costs for each pair of the cities, the authors applied the above-mentioned human migration model to this example. Numerical experiments revealed interesting results concerning the probable equilibria.

The main novelty of the recent paper [7] lies in the proposed definitions of consistent conjectures and in the outskirt of possible ways to calculate the consistent conjectures and the related consistent conjectural variations equilibria (CCVE).

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Motivated by the ideas of bilevel structures of migration processes (the upper-level competition among municipalities, and the lower-level equilibrium among the potential migrants), we propose a new (bilevel) formulation of the human migration model. Under general enough assumptions, we prove the existence of solutions to the bilevel program as well. Preliminary results of numerical experiments (which are underway yet) will be outlined only scarcely.

The paper is organized as follows. Sections 2 through 3, following mainly the previous papers [5]–[7], describe the proposed bilevel human migration model, define the conjectural variations equilibrium at the lower level, and cite some theoretical results from [5] – [6] that establish the existence and uniqueness of the lower level equilibrium as a solution of an appropriate variational inequality problem. The consistency of conjectures and the existence of the corresponding bilevel equilibrium are discussed in Section 4, Unlike the previous paper [9] – [11], here we extend the lower lever (and thus, also the upper level) utility functions from linear to quadratic ones. Conclusion (Section 5), acknowledgments, and the list of references complete the paper.

II. PRELIMINARIES AND PROBLEM STATEMENT

Similar to [4] – [6], consider a closed economy with:

n municipalities (locations), denoted by i
K classes of population denoted by k
$\overline{Q}_i^k$ the initial fixed population of class k in location i
Q_i^k the final population of class k in location i
s_{ij}^k migration flow of class k from origin i to destination j
$c_{ij}^k\left(s_{ij}^k\right) = b_{ij}^k s_{ij}^k + \frac{1}{2} a_{ik}^k \left(s_{ij}^k\right)^2$ cost of migration for group k from location i to location j.

Assume that the migration cost reflects not only the cost of physical movement but also the personal and psychological (affection) cost as perceived by a class when moving between locations. The utility u_i^k (attractiveness of location i as perceived by class k), depends on the population at the destination, that is, $u_i^k = u_i^k\left(Q_i^k\right)$. This assumption is quite natural: indeed, in many cases, the cities with higher population provide much more possibilities to find a job, better medical service and household facilities, a developed infrastructure, etc. On the other hand, when the infrastructure development lags behind the modern city requirements, the higher population may lead to a certain decrease in the living standards (described by certain *utility functions*). These utility functions also incorporate parameters reflecting the scale of investments made by the city's municipality authorities in order to improve the infrastructure, employment capacities, household construction, power supply, and so on. Exactly those sums of investment play the role of the municipality strategies in the game at the upper level.

However, first, we describe the lower level problem. The conservation of flow equations, given for each class k and each location i, and the inequalities forbidding repeated or chain migration, are listed below:

$$Q_i^k = \overline{Q}_i^k + \sum_{j \neq i} s_{ji}^k - \sum_{j \neq i} s_{ij}^k, \qquad i = 1,\dots,n\,, \tag{1}$$

and

$$\sum_{j \neq i} s_{ij}^k \leq \overline{Q}_i^k\,, \qquad i = 1,\dots,n, \tag{2}$$

with $s_{ij}^k \geq 0, \forall k = 1,\dots,K; j \neq i$. Denote the problem's feasible set by

$$M = \left\{ (Q,s) \middle| s \geq 0,\ (Q,s) \text{ satisfies (1) - (2)} \right\}. \tag{3}$$

Equation (1) states that the population of class k at location i is determined by the initial population of class k at location i plus the migration flow into i of that class minus the migration flow out of i for that class. Equation (2) affirms that the flow out of i by class k cannot exceed the initial population of class k at i since no chain migration is allowed.

Assume that the potential migrants are rational and that migration continues until no individual has any intention to the target location and thus any incentive to move since a unilateral decision will no longer yield a positive net gain (the gain in the expected utility value minus the migration cost).

In order to extend the human migration model developed previously in [4], here we introduce the following new concepts.

Definition 1. Let $w_{ij}^{k+} \geq 0$ be an influence coefficient taken into account by an individual of class k moving from i to j. This coefficient is defined by her assumption that after the movement of s_{ij}^k individuals of class k from i to j the total population of class k at j will equal

$$\overline{Q}_j^k + w_{ij}^{k+} s_{ij}^k\,. \tag{4}$$

On the other hand, let $w_{ij}^{k-} \geq 0$ be an influence coefficient conjectured by an individual of class k moving from i to j, determined by the assumption that after the movement of s_{ij}^k individuals, the total population of class k in i will remain

$$\bar{Q}_i^k - w_{ij}^{k-} s_{ij}^k \ . \tag{5}$$

We make the following assumptions concerning the utility functions and expected variations of the utility values:

A1. The utility $u_i^k = u_i^k(Q_i^k)$ is a monotone decreasing and continuously differentiable function.

A2. Each person of class k, when considering her possibility of moving from location i to location j, takes into account not only the difference in the utility values at the initial location and the destination but also both the expected (negative) increment of the value at j

$$s_{ij}^k w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k}, \tag{6}$$

and the expected (positive) affection utility value increment in location i

$$-s_{ij}^k w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} \ . \tag{7}$$

III. Equilibrium Definitions

In this section, we will define what we understand as the conjectural variations equilibrium (CVE) both at the lower and the upper level of the new migration model.

A. Definition of Equilibrium in the Lower Level

Definition 2. A multi-class population and flow pattern $\left(Q^*, s^*\right) \in M$ provide *equilibrium in the lower level*, if for each class $k = 1,\ldots,K$, and for each pair of locations $i, j = 1,\ldots,n;\ i \neq j$, the following relationship holds:

$$u_i^k - s_{ij}^{k*} w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k}\left(Q^*\right) + b_{ij}^k + \\ + a_{ij}^k s_{ij}^{k*} \begin{cases} = u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k}\left(Q^*\right) - \lambda_i^k, & \text{if } s_{ij}^{k*} > 0; \\ \geq u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k}\left(Q^*\right) - \lambda_i^k, & \text{if } s_{ij}^{k*} = 0; \end{cases} \tag{8}$$

and

$$\lambda_i^k \begin{cases} \geq 0, & \text{if } \sum\limits_{l \neq i} s_{il}^{k*} = \bar{Q}_i^k; \\ = 0, & \text{if } \sum\limits_{l \neq i} s_{il}^{k*} < \bar{Q}_i^k. \end{cases} \tag{9}$$

A3. We assume that the influence coefficients are functions depending upon the current population at the location in question and the migration flow from location i to location j, satisfying the following conditions:

$$s_{ij}^k w_{ij}^{k+}\left(Q, s\right) = v_{ij}^{k+} s_{ij}^k + \sigma_{ij}^{k+} Q_j^k \ , \tag{10}$$

and

$$s_{ij}^k w_{ij}^{k-}\left(Q, s\right) = v_{ij}^{k-} s_{ij}^k - \sigma_{ij}^{k-} Q_i^k \ , \tag{11}$$

where

$$v_{ij}^{k\pm} \geq 0,\ \ \sigma_{ij}^{k\pm} \geq 0,\ \ k = 1,\ldots,J;\ \ i \neq j. \tag{12}$$

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Taking into account assumption **A3** and omitting for shortness the argument Q^* in the utility functions, we turn (3) into:

$$u_i^k - s_{ij}^{k*} v_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^{k*} \frac{\partial u_i^k}{\partial Q_i^k} + b_{ij}^k + a_{ij}^k s_{ij}^{k*} = \\ = u_j^k + s_{ij}^{k*} v_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \ \text{if } s_{ij}^{k*} > 0; \tag{13}$$

and

$$u_i^k - s_{ij}^{k*} v_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^{k*} \frac{\partial u_i^k}{\partial Q_i^k} + b_{ij}^k + a_{ij}^k s_{ij}^{k*} \geq \\ \geq u_j^k + s_{ij}^{k*} v_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \ \text{if } s_{ij}^{k*} = 0. \tag{14}$$

Assume that the utility function associated with a particular location and a single class can depend on the population associated with every class and each location, i.e., compose a vector-function $u = u\left(Q\right)$. Assume also that the cost associated with migration between two locations as perceived by a particular class can depend, in general, on the flow of each class between every pair of locations, i.e., compose an aggregate vector-function $c = c\left(s\right)$. Finally, let us compose an auxiliary vector of the appropriate size as follows:

$$d\left(Q, s\right) = \left(d_{ij}^k\left(Q, s\right)\right), \tag{15}$$

where

$$d_{ij}^k\left(Q, s\right) = s_{ij}^k v_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} - \sigma_{ij}^{k-} Q_i^k \frac{\partial u_i^k}{\partial Q_i^k} + \\ + s_{ij}^k v_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} - \sigma_{ij}^{k+} Q_j^k \frac{\partial u_j^k}{\partial Q_j^k}. \tag{16}$$

Now we are in a position to formulate the following result, established in the previous papers [5] – [6]:

Theorem 1. *A population and migration flow pattern* $\left(Q^*,s^*\right)\in M$ *satisfies the equilibrium conditions* (3) *and* (4) *if, and only if it solves the variational inequality problem*

$$\begin{aligned}&\langle -u\left(Q^*\right),Q-Q^*\rangle+\langle c\left(s^*\right)-d\left(Q^*,s^*\right),s-s^*\rangle\ge 0,\\&\forall\left(Q,s\right)\in M.\end{aligned}\tag{17}$$

The existence of at least one solution to the variational inequality (17) follows from the general theory of variational inequalities, under the sole assumption of continuous differentiability of the utility functions *u* and continuity of migration cost functions *c*, since the feasible convex set *M* is compact (*cf.*, for example, [8]).

From now on, we omit the superscript *k* for simplicity purpose. The uniqueness of the equilibrium population and migration flow pattern $\left(Q^*,s^*\right)$ follows under the assumption that the compound operator

$$\begin{pmatrix}-u\left(Q\right)\\ c\left(s\right)-d\left(Q,s\right)\end{pmatrix}: R^{K\times n}\times R^{K\times n\times\left(n-1\right)}\to R^{K\times n}\times R^{K\times n\times\left(n-1\right)},\tag{18}$$

involving the utility and migration cost functions is strictly monotone over the feasible set M:

$$\begin{aligned}&\left\langle\begin{pmatrix}-u\left(Q^1\right)\\ c\left(s^1\right)-d\left(Q^1,s^1\right)\end{pmatrix}-\begin{pmatrix}-u\left(Q^2\right)\\ c\left(s^2\right)-d\left(Q^2,s^2\right)\end{pmatrix},\begin{pmatrix}Q^1-Q^2\\ s^1-s^2\end{pmatrix}\right\rangle>0,\\&\forall\begin{pmatrix}Q^1\\ s^1\end{pmatrix}\ne\begin{pmatrix}Q^2\\ s^2\end{pmatrix},\end{aligned}\tag{19}$$

that is,

$$\begin{aligned}&-\left\langle u\left(Q^1\right)-u\left(Q^2\right),Q^1-Q^2\right\rangle+\left\langle c\left(s^1\right)-c\left(s^2\right),s^1-s^2\right\rangle-\\&-\left\langle d\left(Q^1,s^1\right)-d\left(Q^2,s^2\right),s^1-s^2\right\rangle>0,\forall\begin{pmatrix}Q^1\\ s^1\end{pmatrix}\ne\begin{pmatrix}Q^2\\ s^2\end{pmatrix}.\end{aligned}\tag{20}$$

The following theorem is a consequence of the classical result of the theory of variational inequality problems (*see*, for example, [8]):

Theorem 2. *Consider the variational inequality: Find such* $y^*\in M\subset R^n$ *that,*

$$\left\langle F\left(y^*\right),y-y^*\right\rangle\ge 0,\quad\forall y\in M.\tag{21}$$

If the operator $F: R^n\to R^n$ *is strictly monotone, that is,*

$$\left\langle F\left(y^1\right)-F\left(y^2\right),y^1-y^2\right\rangle>0,\qquad\forall y^1\ne y^2,\tag{22}$$

then the variational inequality (21) *has at most one solution.*

Having established the existence and uniqueness of the (lower level) equilibrium among the potential migrants, we may pass to the concept of the upper level equilibrium among the municipal authorities.

B. Definition of Equilibrium in the Upper Level

We assume that the utility function associated with location *i* and class *k* of potential migrants has the form (which is an extension of the linear utility function used in the previous work [11])

$$u_i^k\left(Q_i^k\right)=A_i^k-\frac{B_i^k}{R_i}Q_i^k-\beta_i^k\left(Q_i^k\right)^2,\tag{23}$$

where $A_i^k>0,B_i^k>0,\beta_i^k>0$ are parameters related to the environment facilities for the potential immigrants of class *k* in location *i*. For instance, the economic sense of A_i^k could be the average cost of a household in location *i* in a district where the typical representatives of class *k* prefer to settle down, while B_i^k might be interpreted as an inverse attraction coefficient for the immigrants of class *k*: that is, the lower value of B_i^k, the higher the degree of attraction revealed by the average family of the specimen of class *k* to the growing population of location *i*. Finally, the parameter $R_i>0$ reflects the amount of investment by the municipal authorities of location *i* into the improvement of the environment for the newcomers and the regular inhabitants: the higher the investment's amount, the lower the negative effect of the growing population on the location's attractiveness for both the current and potential inhabitants.

Now assuming that the investment volumes $R_i>0$ are used as the strategies of the players (municipal authorities of the locations involved), it is standard to define an equilibrium state in the (upper level) game.

Definition 3. An investment vector $R^*=\left(R_1^*,R_2^*,\dots,R_n^*\right)$ is an *equilibrium in the upper level*, if for each location *i*, *i* = 1,…, *n*, the municipal authority's utility function $U_i=U_i\left(R_i,R_{-i}^*\right)$ attains its maximum value exactly at $R_i=R_i^*$ (assuming that all the other players stick to their investment values $R_{-i}^*=\left(R_1^*,\dots,R_{i-1}^*,R_{i+1}^*,\dots,R_n^*\right)$).

Here, the municipality utility function $U_i = U_i\left(R\right)$ is the weighted sum of the location's utility functions of all the classes of potential migrants determined below:

$$U_i\left(R\right) = \frac{Q_i^{1*}}{Q_i^{*}} u_i^1\left(Q^*\right) + \ldots + \frac{Q_i^{K*}}{Q_i^{*}} u_i^K\left(Q^*\right), \tag{24}$$

where Q^* is the equilibrium of the lower level population values, which (due to Theorems 1 and 2) exist uniquely for any (fixed) vector of investments R involved into the structure of locations' attractiveness utility functions (24).

IV. Existence of Bilevel Equilibrium with Consistent Conjectures

The consistency of conjectures (or, the influence coefficients) arises naturally as an important issue. Indeed, the existence of at least one equilibrium for arbitrary influence coefficients obliges one to select some justified conjectures so that the above concept of the equilibrium make sense. In this section, we propose a concept of consistency and formulate the existence result for the consistent conjectural variations equilibrium.

Based on the consistency criterion proposed in [7], we formulate the following definition. Here, for simplicity, we repeat our assumption that the utility function for each location i and each potential migrant group k is quadratic of the form

$$u_i^k\left(Q_i^k\right) = A_i^k - \frac{B_i^k}{R_i} Q_i^k - \beta_i^k \left(Q_i^k\right)^2,$$

with $A_i^k > 0, B_i^k > 0, R_i > 0, \beta_i^k > 0$; next, $a_{ij}^k > 0$ for each quadratic migration cost function; and finally, conjectures (influence coefficients) are constant, i.e., $\sigma_{ij}^{k,\pm} = 0$, and $v_{ij}^{k+} = v_{ij}^{k-} = v_{ij}^k \geq 0,$ for all $i, j, k.$

Definition 4. At a lower level equilibrium (LLE) pattern $\left(Q^*, s^*\right) \in M$, the influence coefficients $v_{ij}^k \equiv w_{ij}^k \frac{B_j^k}{R_j},\ k = 1,\ldots,K;\ i, j = 1,\ldots,n;\ i \neq j$, are referred to as *consistent*, if the following equalities hold:

$$w_{ij}^k = \frac{1}{2\dfrac{B_j^k}{R_j \beta_j^k} + \sum\limits_{\substack{\ell \neq j \\ \ell \neq i}} \dfrac{1}{w_{\ell j}^k + \dfrac{a_{\ell j}^k R_j}{B_j^k}}}. \tag{25}$$

The LLE with consistent conjectures is called *consistent conjectural variations equilibrium* (CCVE) in application to the above-described human migration model.

Now we are in a position to formulate the following existence result.

Theorem 3*. Under assumptions A1, A2, and A3, and if all the investment sums are bounded* (*i.e.,* $0 < R_i \leq R,\ \forall i = 1,\ldots,n$ *), then there exists consistent conjectural equilibrium (CCVE) in application to the above-described bilevel human migration model.*

Proof. Due to paper's size restrictions, all theorems will be proved elsewhere.

When proving Theorem 3, we established that certain finite-dimensional mappings involved in equations (25) are continuous and contracting over corresponding compact subsets. This allows one to find, for each fixed group k of potential migrant and each destination location j, good approximations for the consistent conjectures (influence coefficients)

$$v_{ij}^k \equiv w_{ij}^k \frac{B_j^k}{R_j},\ k = 1,\ldots,K;\ i, j = 1,\ldots,n;\ i \neq j\ , \tag{26}$$

by applying a simple iteration procedure:

$$w_{ij}^{k,(m+1)} = \frac{1}{2\dfrac{B_j^k}{R_j \beta_j^k} + \sum\limits_{\substack{\ell \neq j \\ \ell \neq i}} \dfrac{1}{w_{\ell j}^{k,(m)} + \dfrac{a_{\ell j}^k R_j}{B_j^k}}}, \tag{27}$$

with $w_{ij}^{k,(0)} = 0,\ k = 1,\ldots,K;\ i, j = 1,\ldots,n;\ i \neq j; m = 0,1,\ldots$

Theorem 4. *For each fixed group k of potential migrants and each destination location j, approximate conjectures (influence coefficients)* $v_{ij}^{k,(m)},\ i, j = 1,\ldots,n;\ i \neq j,$ *obtained by formulas* (26) – (27), *converge (as* $m \to \infty$ *) to the unique solution* $v_j^k = \left(v_{ij}^k\right)_{i=1, i \neq j}^{n}$ *of system* (25).

In our future research, we are going to extend the obtained results to the case of not necessarily quadratic affection utility functions and discontinuous conjectures (influence coefficients). However, some of the necessary technique can be developed now, in the case of quadratic utilities and continuous conjectures. To do that, for fixed values of k and j, we denote the value of the inverse of the derivative of the utility function $u_j^k = u_j^k\left(Q_j^k\right)$ by

$$\tau = \left[\frac{du_j^k}{dQ_j^k}\left(Q_j^k\right)\right]^{-1} < 0\,, \tag{28}$$

and rewrite the consistency criterion equalities (25) in a more general form:

$$w_{ij}^k = \frac{1}{-\dfrac{2}{\tau\beta_j^k} + \displaystyle\sum_{\substack{\ell\neq j\\ \ell\neq i}} \frac{1}{w_{\ell j}^k - \tau a_{\ell j}^k}}, \tag{29}$$

where $\tau \in \left(-\infty, 0\right]$. When $\tau \to -\infty$ then the solution of system (29) tends to the unique limit solution $v_{ij}^k \equiv w_{ij}^k \dfrac{B_j^k}{R_j} = 1,\ k = 1,\ldots,K;\ i,j = 1,\ldots,n;\ i \neq j$. For all finite values of the parameter $\tau \le 0$ we establish the following assertion.

Theorem 5. *For each fixed group k of potential migrants and each destination location j, and for any* $\tau \in \left(-\infty, 0\right]$ *, there exists a unique solution of equations* (29) *as a collection of continuous functions* $w_{ij}^k = w_{ij}^k\left(\tau\right),\ i,j = 1,\ldots,n;\ i \neq j$.

Furthermore,

$$w_{ij}^k\left(0\right) = 0, \text{ and } v_{ij}^k\left(\tau\right) \equiv w_{ij}^k\left(\tau\right)\frac{B_j^k}{R_j} \to 1 \text{ as } \tau \to -\infty,$$

for all $i,j = 1,\ldots,n;\ i \neq j; k = 1,\ldots,K.$

V. CONCLUSIONS AND FUTURE RESEARCH

We have investigated a human migration model involving conjectures of the migration groups concerning the variations of the affection utility function values both in the abandoned location and in the destination site. To formulate equilibrium conditions in this model, we use the concept of conjectural variation equilibrium (CVE). We establish the existence and uniqueness results for the equilibrium in question, and introduce a concept of consistent conjectures (influence coefficients), together with the corresponding CVEs. The theorem guaranteeing the existence and uniqueness of the solution to each consistency system and hence, the consistent conjectural variations equilibrium state (CCVE), has been also proved.

We also notice that the human migration model with conjectural variations can be further extended and examined in the case when constraint (2) is replaced by a weaker condition, say

$$Q_i^k \geq 0,\ \ i = 1,\ldots,n;\ k = 1,\ldots,K, \tag{30}$$

which allows us to consider the repeated or chain migration. In this case, the feasible set M stops being compact (remaining, however, convex), which makes insufficient the use of the general theory of variational inequality problems to demonstrate the existence of equilibrium. Then somewhat finer results obtained in [2] – [3] and further developed in [4] can be used to that effect. Indeed, the existence of equilibrium will be guaranteed for various classes of utility functions and migration costs that are free of *exceptional families of elements* (EFE).

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