Feedback Control of a DC Motor: A Hands-on Engineering Approach

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***Abstract*– This paper details real-world implementations of analog controller used for closed loop position control of a DC motor. It shows how to theoretically and experimentally model a DC motor, and how to implement, simulate and test an analog controller in open and closed loop.**

The paper reports on an independent study by an undergraduate electrical engineering student. The idea is to show how educators can enhance student's theoretical knowledge in Control Systems using practical implementation. The reported work can be used as an add-on to existing teaching tools.

I. Introduction

This paper focuses on a comprehensive approach for teaching and learning analysis of a DC motor's in open and closed loop systems. It introduces a practical approach for analysing and understanding the plant in open loop.

The goal of this project is to develop hands-on experience for controlling a DC motor in closed loop. The experience includes:

- theoretical analysis
- mathematical modeling of a real DC motor
- simulation and hardware implementation of analog controllers in open and closed loop.
- This document is separated into four major parts:

Part II & III describe and model the system mathematically. The system is modelled using an open loop transfer function which is later used to simulate open and closed loop designs.

Part IV covers the implementation of analog proportional controllers. After creating analog controllers, the step responses of the actual system are measured and presented side-by-side with the simulated responses. This is done to evaluate the accuracy of the system's modeling.

The paper reports on an independent study by an undergraduate electrical engineering student. The idea is to show how educators can enhance student's theoretical knowledge in Control Systems using practical implementation. The reported work can be used as an add-on to existing teaching tools and textbooks such as [1] [2] [3] [4].

II. System Description

This project uses a hands-on educational tool made by the Feedback Company. This educational system allows for performing experiments of a DC Motor in open and closed loop. The system consists of multiple stages.

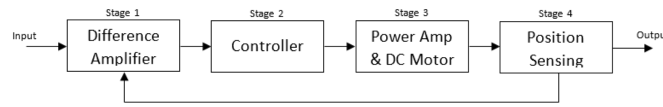

Fig. 1 Feedback System Block Diagram

The first stage is the error amplifier. This unit is simply a difference amplifier. The feedback signal is subtracted from the desired input signal to generate the error signal.

The second stage is the controller. The controller's input is the error signal, and its output feeds the DC motor amplifier. The Feedback system allows for adding an external controller, either analog or digital.

The third stage is the DC motor and its amplifier. The power amplifier is needed to provide adequate power to the DC motor. The fourth stage is the position sensing stage. Here, a rotational potentiometer (connected to the shaft of the DC motor) produces a voltage signal proportional to the angular position of the shaft. This voltage is fed back and subtracted from the input to create the error signal.

Fig. 2 is an open-loop block diagram and transfer function of the DC motor in the s-domain. The transfer function includes gain K and three poles, one of which is an integrator (i.e., located at $s = 0$). The two other poles correspond to the two time constants of the DC motor, i.e., the mechanical and electrical time constants. To quantitatively model the system, the gain and the poles' location must be experimentally obtained (except for the integrator which is an inherent physical relationship between the angular velocity and the angle of the motor).

It is important to note that the measurements made are voltages representing physical quantities. This means that the output of the potentiometer is a measured voltage and is not a measurement of the angle itself. In fact, the angle of the motor's shaft is proportional to the voltage of the potentiometer (for a certain angular range). For this reason, the gain K of the system includes the "angle to voltage" scale factor of the potentiometer.

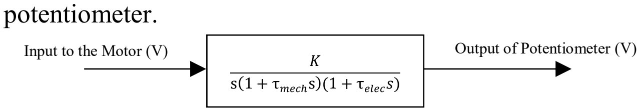

Fig. 2 Open-loop DC Motor Transfer Function

The voltage-to-voltage transfer function of the DC motor is:

$$\frac{Output(s)}{Input(s)} = H(s) = \frac{K_v}{s\cdot(1+\tau_{mech}s)\cdot(1+\tau_{elec}s)} \quad (1)$$

Fig 3 shows the system with a feedback loop with no added controller. It is important to emphasize that when the input voltage and output voltage (or feedback voltage) are equal, the error signal becomes zero. This means there will be a zero voltage input to the motor's amplifier and the system will be stationary.

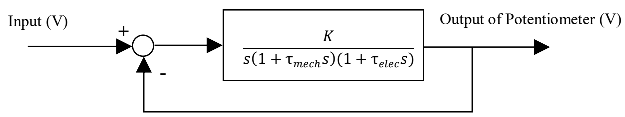

Fig. 3 Closed-loop DC Motor Transfer Function

By incorporating the feedback, the following closed-loop transfer function is obtained:

$$T(s) = \frac{Output(s)}{Input(s)} = \frac{G(s)}{1+G(s)H(s)} \quad (2)$$

$$T(s) = \frac{\frac{K_v}{s\cdot(1+\tau_{mech}s)\cdot(1+\tau_{elec}s)}}{1+\frac{K_v}{s\cdot(1+\tau_{mech}s)\cdot(1+\tau_{elec}s)}} \quad (3)$$

$$T(s) = \frac{K_v}{s\cdot(1+\tau_{mech}s)\cdot(1+\tau_{elec}s)+K_v} \quad (4)$$

III. System Modeling

The objective here is to model the system using the open loop system step response. Fig. 4 below describes the expected result of such an experiment. As can be seen, there are transient response and steady state responses. This allows for identifying some of the DC motor parameters. The output eventually reaches a steady-state slope. This steady state asymptotic line can be characterized using two points (from which the DC motor's parameters can be obtained). If the step input starts at $t = 0$ seconds, then the point where the asymptotic line crosses the horizontal time line is the time constant, i.e., τ. This is based on the assumption that the mechanical time constant is by far larger than the electrical time constant. Therefore, obtaining the mechanical time constant is the more significant approximation.

The system gain, K_v (also referred to as K in the transfer function), is calculated using the slope of the line, m, divided by the step input voltage, V_{in}. Note that K_v has units of $\frac{V/seconds}{V}$ or seconds^{-1}, this is due to the integrator term. $K_v = \frac{\Delta V/\Delta t}{V_{in}} = \frac{V/seconds}{V} = \text{seconds}^{-1}$.

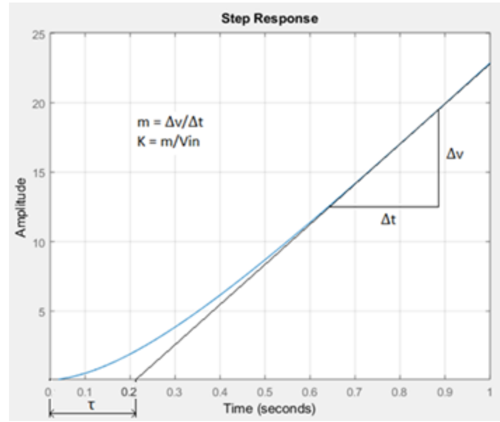

Fig. 4 Step Response used for Gain & Time Constant Calculations

This reiterates that in practice the mechanical time constant is usually by far larger than the electrical one. After experimentally obtaining these values, the transfer function governing the DC motor can then be approximated by:

$$H(s) = \frac{K_v}{s\cdot(1+\tau_{mech}s)\cdot(1+\tau_{elec}s)} \quad (5)$$

From now on τ_{mech} and τ are used interchangeably. Fig. 5 shows the more traditional method of finding the time constant, τ, which is approximated by measuring the time it takes for the step response (shown in blue) to reach 63.2% of its final value. This experiment involves measuring the angular velocity (as opposed to angular position).

The time constant, τ, is determined using the method that was previously described. In this example, the red response (representing the angular position response) is the integration of the blue response (representing the angular velocity response). It too can be used to find the time constant, τ.

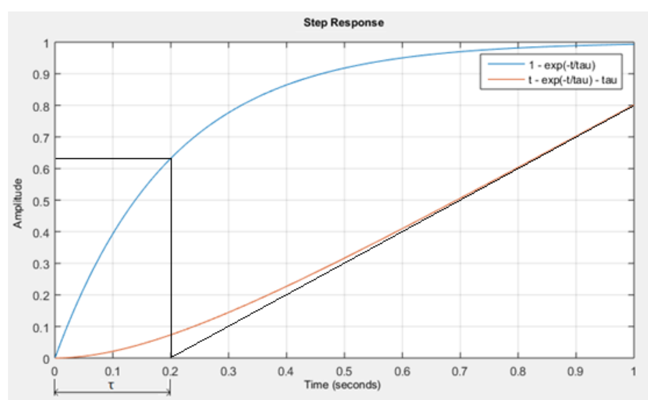

Fig. 5 Step Response with & without an Integrator

The step response (shown in blue) adheres to the following equation:

$$f(t) = 1 - e^{-\frac{t}{\tau}} \quad (6)$$

After integrating this angular velocity function, the angular position is obtained by:

$$\int f(t) = \int \left(1 - e^{-t/\tau}\right) = t + \tau e^{-t/\tau} + C \quad (7)$$

Since at $t = 0$, $\int f(t) = 0$, the integration constant can be obtained as:

$$C = -\tau \quad (8)$$

Adding the integration constant, the following equation is obtained:

$$\int f(t) = \int \left(1 - e^{-t/\tau}\right) = t + \tau e^{-t/\tau} - \tau \quad (9)$$

As t becomes larger, the $\tau e^{-t/\tau}$ terms become negligible and the equation can be approximated by:

$$\int f(t) \approx t - \tau. \quad (10)$$

This simple equation can then be used to approximate the time-constant, τ_{mech}.

Fig. 6 shows an oscilloscope image taken during the modeling experiments. The orange line shows the step input waveform and the blue line shows the potentiometer's output waveform.

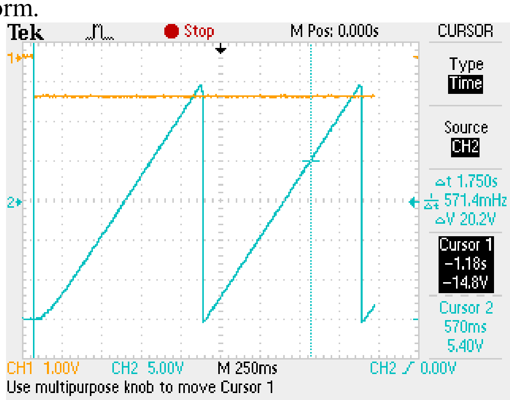

Fig. 6 Potentiometer Step Response

Note: for a step input, the motor will eventually reach a steady-state value of the angular velocity due to the inherent relationship between the angular velocity and angle itself. Then, the steady-state angular velocity is equal to the steady-state slope of the angular position step response.

The table below shows data collected during the modeling experiments. Table 1 shows the results of multiple experiments. Each row in Table 1 corresponds to a specific experiment using a different input voltage. For each experiment, two points were taken from the scope that corresponds to the voltage of the potentiometer, Shown as $V_{pot\,1}$, $Time\ 1$ and $V_{pot\,2}$, $Time\ 2$, respectively.

The reason for the redundancy, is to a derive more accurate values for τ and K.

TABLE I

DATA COLLECTED FROM OSCILLOSCOPE

V_{in} Volts	$V_{pot\,1}$ Volts	Time 1 sec	$V_{pot\,2}$ Volts	Time 2 sec
2.021	21.80	0.508	27.80	0.612
1.577	47.60	1.150	55.80	1.330
2.283	46.00	0.900	55.60	1.060
1.833	42.00	0.910	53.20	1.120
1.510	41.60	1.050	52.80	1.300
1.356	38.80	1.080	49.60	1.350
1.072	37.40	1.260	48.20	1.590
0.818	41.40	1.740	51.00	2.120
0.594	43.20	2.400	53.40	2.940

For each experiment, the slope of the graph, the gain and the x & y intercepts were calculated and are shown in Table 2. The gain was calculated by dividing the slope by the input voltage.

TABLE II

CALCULATED SYSTEM PARAMETERS

m – slope Volts/sec	K – gain sec^{-1}	B – y-int Volts	τ – x-int sec
57.692	28.546	-7.508	0.130
45.556	28.887	-4.789	0.105
60.000	26.281	-8.000	0.133
53.333	29.096	-6.533	0.123
44.800	29.669	-5.440	0.121
40.000	29.499	-4.400	0.110
32.727	30.529	-3.836	0.117
25.263	30.884	-2.558	0.101
18.889	31.799	-2.133	0.113

Table 3 shows the average gain, K, and the average time constant, τ, calculated using the data in Table 2.

TABLE III

AVERAGE SYSTEM PARAMETERS

K average Volts	τ average seconds
29.74	0.108

These calculated averages are then used to define a transfer function which describes the system. Note: τ_{elec} was calculated during an experiment using frequency response analysis which is not mentioned in this paper. ω_{elec} was obtained by observing the high frequency response of the open loop DC motor where the slope changes by -20 dB/dec.

$$H(s) = \frac{K}{s \cdot (1 + \tau_{mech} s) \cdot (1 + \tau_{elec} s)} \quad (11)$$

Since $\omega_{mech} = \tau_{mech}{}^{-1}$, the equation can also be written as:

$$H(s) = \frac{K}{s \cdot \left(1 + {s}/{\omega_{mech}}\right) \cdot \left(1 + {s}/{\omega_{elec}}\right)} \quad (12)$$

Using the values from Table 3:

$$H(s) = \frac{29.74}{s \cdot (1 + 0.108 \cdot s) \cdot \left(1 + {s}/{\omega_{elec}}\right)} \quad (13)$$

The time constant, τ, can be written as a frequency:

$$\omega_{mech} = \frac{1}{\tau_{mech}} = \frac{1}{0.108\ \text{sec}} = 9.26\ \frac{\text{rad}}{\text{sec}} \quad (14)$$

Using the data collected in the experiments, a transfer function that describes the overall system can be formulated. The electrical and mechanical poles that were experimentally obtained can be written as:

$$\omega_{mech} = 9.26\ \frac{\text{rad}}{\text{sec}} \tag{15}$$

$$\omega_{elec} = 275\ \frac{\text{rad}}{\text{sec}} \tag{16}$$

From Table 3, the voltage-to-voltage gain is:

$$K = 29.74\ \frac{\text{V}}{\text{V}} \tag{17}$$

Combing these terms results in the open-loop transfer function shown below:

$$H(s) = \frac{K}{s\left(1 + \frac{s}{\omega_{mech}}\right)\left(1 + \frac{s}{\omega_{elec}}\right)}\ \frac{\text{V}}{\text{V}} \tag{18}$$

Substituting the above values:

$$H(s) = \frac{29.74}{s\left(1 + \frac{s}{9.26}\right)\left(1 + \frac{s}{275}\right)}\ \frac{\text{V}}{\text{V}} \tag{19}$$

IV. Implementing an Analog Controller

Fig. 7 shows how a proportional controller can be implemented using op-amp configurations. The input signal of the controller is the error signal, and the output signal of the controller is fed to the DC motor's amplifier.

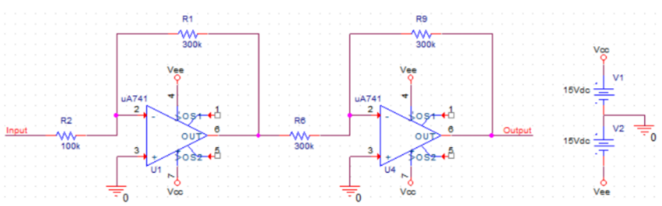

Fig. 7 Proportional Analog Controller

The following experiments show the effect of different proportional controllers on the closed-loop system. Both simulated (blue) and experimental (red) data are shown. The P term is different in each case to show its effect on the system's step response.

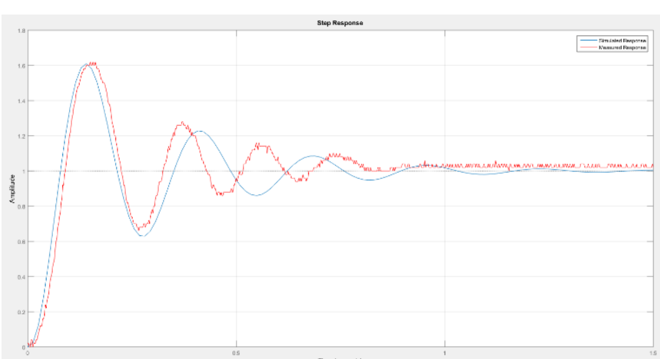

Fig. 8 Proportional Controller Simulated & Measured Step Response (Kp=2)

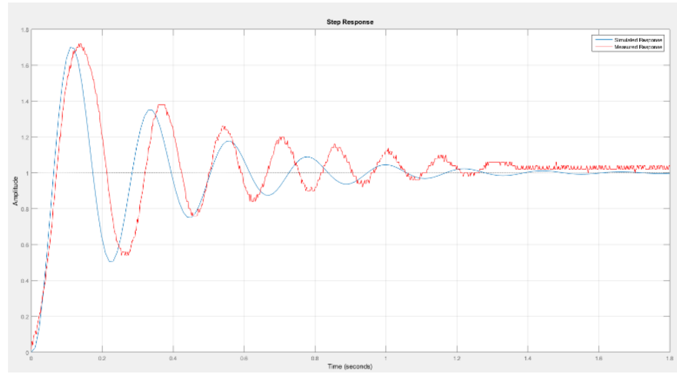

Fig. 9 Proportional Controller Simulated & Measured Step Response (Kp=2)

Note that as K_p increases, the step response appears to have a larger overshoot and a longer settling time. This means that with higher K_p, the system is also closer to instability.

V. Conclusion

This paper reports on a hands-on experience for controlling a DC motor. It was part of a Direct Independent Study (DIS). The student gained real experience as it relates to system modeling and analog controllers. The student gained theoretical and practical understanding of open and closed loop control issues, some of which are not even mentioned in Control Systems textbooks.

These kinds of hands-on projects can benefit both professors and students, since they complement the math-loaded Control classes. They add relevance to the topics of modelling and design.

VI. References

[1] Åström, K. J., & Wittenmark, B. (1984). *Computer controlled systems: Theory and design* (Third ed.). Englewood Cliffs, NJ: Prentice-Hall.

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[4] Dorf, R. C., & Bishop, R. H. (2001). Modern control systems (Tenth ed.). Upper Saddle River, NJ: Prentice Hall.FF